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Investigation of chaotic dynamics in variable stars

Theses of dissertation

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1. Background

Pulsating variables may show chaotic behaviour that is caused by non-linear coupling of radial modes. Chaos was not expected in RR Lyrae stars before, but recent space photometric observations have revealed the signs of non-linear dynamics in these stars. The phenomenon of period doubling bifurcation is clearly visible in the data of *CoRoT* and *Kepler* space telescopes. A slight change in the parameters of these dynamical systems can lead to chaos through a bifurcation cascade. Hydrodynamical models confirmed that this is possible in RR Lyrae stars. According to the theory, period doubling is caused by the instability of the 9:2 resonance between the fundamental mode and its ninth overtone. However, the presence of the first overtone is also required to evolve to chaos. The 9:2 resonance itself may be responsible for the mysterious amplitude and phase modulation, the Blazhko effect, as well. If the explanation is right, the irregular nature of the Blazhko effect must be chaotic. The detection of chaos is strongly limited by the quality of observational data. Dynamical investigations require long, continuous and extremely precise data. In the case of RR Lyrae stars it can be achieved only from space.

My PhD work is focused on the detectability of chaotic dynamics of variable stars. The thesis is divided into three topics: testing the global flow reconstruction method that is able the determine the quantitative properties of the dynamics, the dynamical analysis of RR Lyrae hydrodynamical models, and the investigation of the possible chaotic nature of the Blazhko effect in the case of a *Kepler* star.

2. Methods and Data

I used the global flow reconstruction method in the dynamical investigations. This is a nonlinear analyser tool that is able to detect low dimensional chaos, and determine the fractal dimension (Lyapunov dimension) of the system. The method is based on the search of the nonlinear map that connects the neighbouring points of the phase space trajectory. Once we have found the map, an arbitrarily long synthetic data can be iterated. The long dataset is required to determine the rate of the exponential divergence of the chaotic trajectories. It is practical to use a large parameter space, where we can found numerous synthetic signals that can be compared to the original data. For comparison different representations are useful: the Fourier spectrum, the Broomhead-King projections and the data set itself. The quantitative properties of synthetic data are good representations of the original data.

For testing the global flow reconstruction method I generated suitable data sets of chaotic coupled Rössler oscillators. The Rössler oscillators have been already found to be useful in previous tests of the method, and their cycles are similar in shape to the cycles of pulsating stars. Observational data often contain long-term variations that is not caused by pulsation. I tested two methods that are able to eliminate these variations: the EMD method (Empirical Mode Decomposition) and the Fourier filtering. The basic difference between the methods is not only the empirical and the analytical approach, but that the EMD process is optimized to handle the time series itself while the latter is designed to be applied to the Fourier transform.

I also investigated chaotic RR Lyrae hydrodynamical models. The models are taken from the recently generated model family of the Florida-Budapest code. The code calculates one-dimensional, turbulent convective pulsation models.

My most recent dynamical analysis is focused on the Blazhko-star V783 Cyg that has the shortest amplitude modulation cycles in the original *Kepler* field. The most suitable data for the analysis is the recently published tailor-made aperture photometry of the star. I separated the amplitude and phase modulations from the light curve using the pulsation amplitude maxima and time dependent Fourier parameters derived by the analytical function method.

3. Theses

Testing the global flow reconstruction method:

1. **The errors of the frequency determination of the reconstruction do not correlate with the Lyapunov dimension of synthetic signals.**

   I investigated whether any relationship can be detected between the accuracy of reconstructed frequencies and the Lyapunov dimensions that refer to the complexity of the system. The relationship of the uncertainties of these quantities would provide objective criteria in the comparison of the original and synthetic data sets that would also speed up the process. I used coupled Rössler oscillators in the test . The results showed that the frequency shift is independent of the quantitative parameters. [1]

2. **Elimination of long-term variation from the data sets is helpful in the dynamical investigations.**

   Observed light curves may show long-term variations that are not caused by pulsation. These variations affect the nonlinear dynamical investigations. I tested two methods that are able the eliminate these variations: the Fourier filtering and the EMD method. I demonstrated that the elimination of these variations makes it easier to detect chaotic dynamics. [2]

Dynamical investigation of RR Lyrae models:

3. **The Lyapunov dimension of the analysed chaotic RR Lyrae models are** ~ 2.2 **that can be derived from both the radius variation and the luminosity.**

   I performed the global flow reconstruction of four chaotic hydrodynamic models calculated by the Florida-Budapest code. I derived the Lyapunov dimensions to be ~ 2.2 in all cases. The most suitable global parameter for dynamical analysis is the radius variation. The luminosity that can be directly observed from the stars is the result of complex phenomena and therefore it is difficult to investigate. In spite of the difficulties, my reconstruction was successful, but the results are less robust than in the case of radius variation. The transformation of the luminosity into a simpler form can be useful in the dynamical analysis. [3]

4. **The 6:8 resonance detected in the star RR Lyr can be found in the dynamical neighbourhood of the RR Lyrae models.**

   The frequency-shift property of the global flow reconstruction provides the opportunity to construct synthetic signals, dynamically similar to the original data, that approach resonance. This is a useful feature to extend the RR Lyrae models in studying nonlinear dynamical phenomena. I found synthetic signals approaching a resonance (6:8) that was not detected in hydrodynamic models but it is present in observational data. [3]

Dynamical analysis of the Blazhko modulation:

5. **The assumption of the chaotic nature of the Blazhko effect cannot be confirmed by the dynamical analysis of V783 Cyg.**

   I performed a nonlinear analysis on the Blazhko effect for the first time. I used the data of V783 Cyg that has the shortest amplitude modulation period among the RR Lyrae stars observed by the *Kepler* space telescope and therefore it was found to be the most suitable for the analysis. The separation of the modulation curves from the light curve was affected by data processing problems. The modulation of this star can be fitted by chaotic signals, however, due to the strong interference of the instrumental and data processing effects, this result is not reliable. [4]

4. Conclusions

The dynamical analysis of classical variable stars is a future scientific program that may have a great impact. We can achieve more and better hydrodynamical models with the increasing computing capacity. There are already indications that nonlinear dynamics play an important role in these classical pulsators. This will be also detectable from more suitable observational data. The goal of my investigation was to find out how the global flow reconstruction method can be applied in this topic and what are its limitations.

I showed that the method is useful for both the modelling and the investigation of observational data. The frequencies and the dynamical properties of the synthetic signals of the reconstruction do not show any relationship. This frequency shift property can be considered as a useful feature of the method that makes it able to search for resonances in the dynamical neighbourhood of the system. We can find resonances that the hydrodynamical models cannot produce due to their limitations. Knowing the possible resonances helps to find them in observational data.

The shape of the light curves of RR Lyrae star are complex, their fine structures are formed by shock waves. The method does not reconstruct the fine structures, only the main characteristics of the light curve. My investigation of RR Lyrae models showed that the quantitative properties of the dynamics can be derived not only from the radius variation but from the luminosity variation, as well. Thus light curves are useful in the search of chaotic dynamics.

The quality of the data is crucial in the analysis. Instrumental effects, like gaps, trends and data stitching problems due to the sensitivity differences of the CCD detectors can interfere with the dynamical investigations. Sufficient sampling is also important to obtain reliable results. Currently, suitable light curves can be provided only by the 1 minute sampling of the *Kepler*. These requirements are included in our proposals for targets of the extended (*K2*) mission: the most promising irregular targets are requested in short cadence mode.

Irregularities are observed in modulated stars. The modulation itself probably affects the global flow reconstruction. My test showed that the long-term variations cannot prevent the successful reconstruction, but their elimination helps significantly. That can be true for modulation, as well. According to my experiences, the separation of the modulation from the light curve is complicated and unreliable. It is probably more practical to perform the analysis without the elimination of the modulation. The investigation of the modulation itself did not yield unambiguous results in the case of V783 Cyg. Two more RR Lyrae stars are suspected to be useful in dynamical analysis in the original *Kepler* field. Future space missions, like *TESS* and *PLATO* may also provide suitable candidates.

5. The theses are based on the following refereed publications

[1] Plachy, E., Kolláth, Z.: *Testing the global flow reconstruction method on coupled chaotic oscillators*, 2010, JPhCS, 218, 012029

[2] Plachy, E., Kolláth, Z.: *Elimination of long-term variations from chaotic light curves*, 2013, AN, 334, 984

[3] Plachy, E., Kolláth, Z., Molnár, L.: *Low-dimensional chaos in RR Lyrae models*, 2013, MNRAS, 433, 3590

[4] Plachy, E., Benkő, J. M., Kolláth, Z., Molnár, L., Szabó, R.: *Non-linear dynamical analysis of the Blazhko effect with the Kepler space telescope: the case of V783 Cyg*, 2014, MNRAS, 445, 2810