Lee Yang Model in Presence of Defects

Excerpts of the PhD Thesis

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1 Introduction

The study of the Lee-Yang model is important for a general understanding of two dimensional integrable models. The main motivation behind that comes from AdS/CFT duality. In our quest for understanding realistic but complicated models like the Super Yang-Mills gauge theories, there is an important conjecture called the AdS/CFT duality and it states the equivalence of $\mathcal{N} = 4$ Super Yang-Mills gauge theory with superstrings on $AdS_5 \times S^5$.

In this thesis I choose the Lee-Yang model and go through different approaches to analyze the model using the form factor approach and the bootstrap program, the lattice and the TBA equations from the lattice as different approaches that lead to a full picture about the model.

The bootstrap program aims to classify and explicitly solve 1+1 dimensional integrable quantum field theories by constructing all of their Wightman functions. In the first step, called the S-matrix bootstrap, the scattering matrix, connecting asymptotic in and out states, is determined from its properties such as factorizability, unitarity, crossing symmetry and Yang-Baxter equation (YBE) supplemented by the maximal analytical assumption.

In developing a defect form factor program the first step is the T-matrix bootstrap. Interacting integrable defect theories are purely transmitting and topological. As a consequence a momentum like quantity is conserved and the location of the defect can be changed without affecting the spectrum of the theory. This fact, together with integrability lead to the factorization of scattering amplitudes into the product of pair-wise scattering and individual transmission and enable one to determine the transmission factors from defect YBE (DYBE), unitarity and defect crossing unitarity. The second step is the defect form factor bootstrap: Once the transmission factors are known we can formulate the axioms that have to be satisfied by the matrix elements of local defect operators. We will analyze both operators localized in the bulk and also on the defect. By finding their solutions the spectral representation of any correlator can be determined and theory can be solved completely.

Thermodynamic Bethe Ansatz (TBA) equations have been introduced as an important tool in the study of both massive and massless integrable quantum field theories. The Lee-Yang model was studied from the TBA approach.

However, the lattice approach is far more reaching. It is a systematic approach that allows to obtain both massive and massless excited TBA equations by studying the continuum scaling limit of the associated integrable lattice models. The most important input from the lattice approach is an insight into the analytic structure of the excited state solutions of the TBA equations. Previously this structure had to be guessed.

In this thesis we turn our attention to the simplest example of a non-unitary minimal theory $\mathcal{M}(p, p')$ with $|p - p'| \neq 1$, namely, the Lee-Yang minimal model $\mathcal{M}(2, 5)$.

We study the Lee-Yang model on the lattice. We analyze the periodic, boundary and the seam cases in both massive and massless regimes. We derive their ground state TBA and analyze the flows from the $(r, s) = (1, 1)$ to the $(r, s) = (1, 2)$ sectors in both boundary and seam (defect) cases.

In the large volume limit, I calculate the Luscher correction terms of the TBA energy of a one-particle state to higher orders. This generalization is important as similar techniques are used in the analysis of energy corrections due to wrappings for superstrings in the sigma model. I display those ideas in the end of the presentation as this is an important direction of work in string theory models, particularly in the AdS/CFT correspondence.

2 Objectives

In the course of work on this thesis, I had the objective of analyzing some important aspects of the Lee-Yang model, especially in presence of defects. The main ideas that were of special interest to analyze and understand are the following:

1. Determine the bulk form factors of the model in presence of defects, in the two cases where the operators are on the same side or on different sides of the defect.

2. Determine the defect form factor solutions using the kinematic and the dynamic recursion relations for the primary fields as well as their first and second order descendants.

3. Try to establish a one-to-one relation between the defect form factor solutions of the primary fields and their descendants with the Hilbert space of the model.

4. Establish a general method to use the form factors of a model with a boundary in order to derive the form factors of the same model with another boundary condition using the transmission matrix of that model.

5. In the lattice approach I wanted to study the flows in the boundary case from the $|0\rangle \to |\Phi\rangle$ modules.

6. Use the lattice to analyze and classify the more general case of boundary flows of the Lee-Yang model in presence of a seam, regarding their paths, zero structures and their states in the Hilbert space.

7. Use the transfer matrices of the periodic, boundary and seam cases from the lattice to analyze their respective analytic structures.

8. Determine the TBA equations for the periodic, boundary and seam Lee-Yang models in the massless and massive regimes using the information of their analytic structure.

9. Determine the Luscher energy corrections to the Bethe-Ansatz energy of the Lee-Yang model, and give a generalization of this calculation to sigma superstring models used in the AdS/CFT correspondence studies.

3 Results

3.1 Form Factors in presence of defects:

3.1.1 Bulk form factors

I calculate the bulk form factors and the bulk two point functions in the presence of a defect int the Lee-Yang model for operators on same side and on opposite sides of the defect.

For operators on same side of the defect: If both operators are localized on same side of the defect then the elementary form factors are the same as the bulk form factors B_n of the infinite volume model without the defect. We can conclude that the two point function in this case is *exactly the same* as the bulk two point function.

For operators on same side of the defect: If the operators are localized on different sides of the defect the two point function is expressed in terms of the bulk form factors B_n and the transmission matrix $T_-(\theta)$, due to the presence of the defect between the operators. The additional contribution caused by the defect is the insertion of the product of the transmission matrices of each particle rapidity in the expression of the bulk two point function.

3.1.2 Defect Form Factors

Using the kinematic and the dynamic recursion relations derived from the defect form factor axioms, I determine the solutions for the primary fields and their first order and second order descendants. They are expressed in terms of the elementary symmetric polynomials σ.

Form factor solutions of primary fields The primary fields of the model on the defect are five. The identity I, the limiting cases of the bulk fields living on the left and the right of the defect Φ_+ and Φ_-, in addition to the two chiral fields φ and $\bar{\varphi}$ appearing solely on the defect. Using the recurrence relations, we are able to provide their form factor solutions up to levels one and two.

First Order Descendants Using same recursion relations we are able to determine form factor solutions that can be identified as the first order descendants of the primary fields by dimensional analysis and satisfaction of recursion relations. Those solutions correspond to the fields: $\partial\Phi_+$, $\partial\Phi_-$, $\bar{\partial}\Phi_+$, $\bar{\partial}\Phi_-$, $\partial\varphi$ and $\bar{\partial}\bar{\varphi}$.

Also the second order descendants were calculated for the those operators together with the identity operator. The solutions are shown in tables summarizing all primary field solutions and their first and second order descendants.

3.1.3 Boundary form factors via defects

I illustrate how defects can be used to generate new boundary form factor solutions from those of previously solved boundary conditions. The underlying fusing idea for the reflection matrices can be explained as follows: Suppose we place an integrable defect with transmission factor $T_-(\theta)$ in front of an integrable boundary with reflection factor $R(\theta)$ and boundary form factor $F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_n)$. We claim that the fused form factor $\bar{F}_n^{\mathcal{O}}(\theta_1, \ldots, \theta_n)$ calculated from the product of the $T_-(\theta_i)$ and $F_n^{\mathcal{O}}(\theta_1, \ldots, \theta_n)$, satisfies boundary form factor axioms of the fused boundary (from the defect and the original boundary) with reflection matrix $\bar{R}$ that can be calculated from T_- and R.

3.2 The Lee-Yang on the lattice

3.2.1 Flows between boundary conditions:

In terms of the Virasoro modes: all the highest weight states flow to each other $|0\rangle \to |\Phi\rangle$ and in the module the rule is very simple we have to increase the index of every Virasoro mode by one $L_{-n} \to L_{-n+1}$:

$$L_{-n_1}L_{-n_2}...L_{-n_k}|0\rangle \to L_{-n_1+1}L_{-n_2+1}...L_{-n_k+1}|h\rangle$$

I provide tables and plots showing the flow from each state to its corresponding state in the other module. Same applies on the paths and zero structure.

3.2.2 Flows in case of a seam

Introducing a seam we analyze the two limiting cases namely ξ (the seam parameter) going from 0 to ∞. In the $\xi \to \infty$ limit we found the following identification between the zero distribution, the paths and the Virasoro modes. The flows are explained in three mechanisms:

1. If the outermost string is a 1-string, it flows towards infinity with increasing ξ. The corresponding flow of states is:

$$L_{-N_1}...L_{-N_n}\bar{L}_{-\bar{N}_1}....\bar{L}_{-\bar{N}_n}|0> \to L_{-N_1}...L_{-N_n}\bar{L}_{-\bar{N}_1+1}....\bar{L}_{-\bar{N}_n+1}|\bar{\phi}>$$

with $|0> \to |\bar{\phi}>$

2. If the outermost string is a short 2-string, one of the zeroes flows to

infinity and the other goes to $\frac{3\pi}{10}$. The corresponding flow of states is:

$$(L_{-N_1}...L_{-N_n}\bar{L}_{-\bar{N}_1}....\bar{L}_{-\bar{N}_n})\bar{L}_{-1}|\Phi > \to (L_{-N_1}...L_{-N_n}\bar{L}_{-\bar{N}_1+1}....\bar{L}_{-\bar{N}_n+1})|\phi >$$

3. If the outermost string is a long 2-string, it flows towards infinity. The corresponding flow of states is:

$$L_{-N_1}...L_{-N_n}\bar{L}_{-\bar{N}_1}....\bar{L}_{-\bar{N}_n}|\Phi > \to L_{-N_1}...L_{-N_n}\bar{L}_{-\bar{N}_1+1}....\bar{L}_{-\bar{N}_n+1}|\Phi >$$

Those results are pictorially shown in representative plots.

3.2.3 TBA equations for the Lee-Yang

The TBA equations of the Lee-Yang are solved for the periodic boundary conditions with and without a seam and also for the boundary case, in both massive and massless regimes. The TBAs are solved for $(r, s) = (1, 1)$ and $(r, s) = (1, 2)$ sectors in the continuum limit. The TBAs are integral functional relations for the normalized eigenvalues of the transfer matrices of the model. They can be used to deduce the ground state as well as excited state energies of the model.

3.3 Luscher corrections

For a single impurity operator, for the simplest nontrivial $\mathcal{N} = 1$ supersymmetric β-deformed theory with $\beta = \frac{1}{2}$, I evaluate the Lüscher corrections of the energy at NNLO in the coupling. It is shifted as

$E = E_{ABA} + \Delta E$. The asymptotic Bethe Ansatz energy E_{ABA} is the dispersion relation of a standing particle while ΔE corresponds to the wrapping interactions. The lowest order and the next-lowest order corrections were calculated for this model and I evaluated the next-next-lowest order correction and obtained a numerical value in terms of the multiparticle zeta functions ξ. This correction turns out to be:

$$\Delta E_{NNLO} = -128(-192\zeta(3) + 112\zeta(3)^2 - 592\zeta(5) - 440\zeta(3)\zeta(5) - 322\zeta(7) + 1701\zeta(9))$$

4 References

The original results presented in the dissertation were based on the papers:

- Zoltan Bajnok, Omar el Deeb: *Form factors in the presence of integrable defects*, Nuclear Physics B 832, p. 500-519, 2010, arXiv:0909.3200 [hep-th]

- Zoltan Bajnok, Omar el Deeb, Paul Pearce: In Progress

- Zoltan Bajnok, Omar el Deeb: *6-loop anomalous dimension of a single impurity operator from AdS/CFT and multiple zeta values*, JHEP 1101:054,2011, arXiv:1010.5606 [hep-th]