PhD Thesis

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Thesis Title: Non-extensive Statistical Physical Methods in the Interpretation of High-energy Particle Spectra

PhD School: Doctoral School of Physics, Eötvös Loránd University

PhD Programme: Particle Physics and Astronomy

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November, 2012.

Introduction

One of the main findings of the RHIC (Relativistic Heavy-Ion Collider at Brookhaven) and LHC (Large Hadron Collider near Geneva) are the experimental indications for the formation of quark-gluon plasma (QGP) in high-energy heavy-ion (AA) collisions. The formed QGP is very short lived, thus its properties can be examined only indirectly through the distributions of hadrons (spectra, angular and rapidity correlations, nuclear modification factor R_{AA}, multiplicity distributions) forming from it. In order to obtain information on the properties of the QGP from the measured hadronic observables, it is essential to understand the non-perturbative mechanism of hadronisation. My achievements in the description of hadronisation can be summarised in the following points:

- The application of non-extensive thermodynamical and statistical physical methods in the description of particle spectra produced in high-energy heavy-ion, proton-proton and electron-positron collisions.
- A super-statistical modelling of the process of jet-fragmentation, developement of new functional formulas in both the canonical and microcanonical framework.
- Proof for the new theoretical models to be in good agreement with LEP, RHIC and LHC measurements.

For the list of the publications of these results see the ***List of Publications*** section.

Thesis Points

In the literature, fragmentation and coalescence models are used for describing hadronisation. The results I present below contain two types of coalescence and one fragmentation model.

I. Hadronisation Via Microcanonical Fragmentation

In high-energy physics, fragmentation functions are most frequently used for describing hadronisation. These are phenomenologically parametrised functions based on data measured in electron-positron (e^+e^-) collisions. Though the scale dependence of these functions can be calculated in perturbative quantum chromodynamics (pQCD), there is no theoretical basis for their actual form at a certain scale. In papers [1, 2, 3, 4], I have shown that an excellent description of fragmentation functions measured in e^+e^- and proton-proton (pp) can be achieved by a microcanonical fragmentation model (see **Fig. 1**). This model is based on two assumptions: *i,* the cross-section of the production of hadrons inside a jet is proprtional to the phase space of the hadrons constrained *only* by energy conservation (thus hadrons inside a jet form a microcanonical ensemble); *ii,* hadron-multiplicity in a jet fluctuates according to the Euler-gamma distribution.

In paper [6], it is shown that when properly parametrised, the low-energy (canonical) approximation (Tsallis-Pareto distribution) of the obtained analitical result satisfies the DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) scale evolution equations derived from QCD. Furthermore, in a pQCD improved parton model calculation, qualitatively similar pion spectra may be obtained using the Tsallis-Pareto type fragmentation function and the AKK (Albino–Kniehl–Kramer - type), the most frequently used one in the literature.

II. Hadronisation Via Direct Coalescence

pQCD improved parton model calculations, in which fragmentation functions are used, reproduce hadronspectra measured in AA collisions only for $p_T \gtrapprox 4-7$ GeV/c due to the breakdown of perturbation theory and the emergence of collective effects at low energy. For the description of the low-energy part of the spectra of hadrons stemming from AA collisions, thermal coalescence models turned out to be useful. To obtain a colaescence model valid for an energy range stretching up to where parton model calculations become valid, it is essential to generalise the Boltzmann-Gibbs thermodynamics. In practice, the Tsallis-Pareto distribution has proven to be the most efficient. In the models published in papers [8, 10-13], hadrons are formed via rapid coalescence of thermal quarks; the collective flow of the QGP is accounted for using the *Blast Wave* model; this coalescence model is energetically additive. Using the Tsallis-Pareto distribution for the energy distribution of the quarks, the calculated π, K, η, ϕ, p, Ξ, Λ spectra agree with RHIC measurements within 40% errors for transvers hadron momenta $p_T \lessapprox 5-6$ GeV/c (see **Fig. 2, left panel**), while using the Boltzmann-Gibbs distribution, the calculations give good agreement with measurements only for $p_T \lessapprox 2-3$ GeV/c. Furtheremore, it is also interesting that for the momenta $p_T \gtrapprox 0.5$ GeV/c (the experimentally measurable range), hadron spectra are rather insensitive to whether the hadronisation (freeze-out) hypersurface is space- or time-like.

In paper [7], I have shown that the Tsallis-Pareto distribution fits excellently to transverse spectra of charge averaged hadrons produced in pp collisions at $\sqrt{s} \in$ [200 GeV, 7 TeV] energies (see **Fig. 2, right panel**).

III. Hadronisation of a Non-extensive Quark Matter Via Resonance Production

Tsallis-Pareto equilibrium single-particle distribution emerges in a system the total energy E of which is related to the single-particle energies ϵ_i through the composition rule $E = \epsilon_1 + \cdots + \epsilon_N + a(\epsilon_1\epsilon_2 + \cdots + \epsilon_{N-1}\epsilon_N) + \cdots + a^{N-1}\epsilon_1 \ldots \epsilon_N$ [5, 9, 11]. In [5], I have shown that both the mass distribution of pre-resonances produced by the coalescence of such quasi-quarks and the spectrum of pions stemming from the decay of these resonances show power-law behaviour (see **Fig. 3**).

In jets produced in e^+e^- or in pp collisions, generally only a few dozen particles are produced, whose energy is often of the same order of magnitude as that of the total energy of the jet. Thus, for practical purposes, I also calculated the microcanonical single-particle distribution in a system described by the interactions above [2]. The comparison of the analytic calculation with simulated results and measurements is shown in **Fig. 4**.

List of Publications

[1] K. Urmossy, G. G. Barnaföldi, T. S. Biró, Microcanonical Jet-fragmentation in proton-proton collisions at LHC Energy
Phys. Lett. B, **718**, 125-129, (2012)

[2] K. Urmossy, T. S. Biró, G. G. Barnaföldi, Generalised Microcanonical Statistics and Fragmentation in Electron-Positron Collisions
Acta Physica Polonica B, **5**:(2), pp. 363-368, (2012)

[3] T.S. Biró, K. Urmossy, P. Ván, G.G. Barnaföldi, Z. Schram, Non-extensive statistical model for strange and non-strange hadron spectra at RHIC and LHC energies
Acta Physica Polonica B, **43**:(4) pp. 811-820, (2012)

[4] K. Urmossy, G. G. Barnaföldi, T. S. Biró, Generalised Tsallis Statistics in Electron-Positron Collisions
Phys. Lett. B, **701**, 111-116, (2011)

[5] K. Urmossy, T. S. Biró, G. G. Barnaföldi, Pion Production Via Resonance Decay in a Non-extensive Quark-Gluon Medium with Non-additive Energy Composition Rule
EPJ Web of Conferences, **13**: 05003 (2011)

[6] G. G. Barnaföldi et. al, Tsallis-Pareto like distributions in hadron-hadron collisions
Gribov 80 - Memorial Volume, Singapore: World Scientific, 2011. p. 357. (ISBN: 978-981-4350-18-1), (2011)

[7] G. G. Barnaföldi, K. Urmossy, T. S. Biró, Tsallis-Pareto like distributions in hadron-hadron collisions
J. Phys. Conf. Ser. **270** 012008, (2011)

[8] K. Urmossy, T. S. Biró, Cooper-Frye Formula and Non-extensive Coalescence at RHIC Energy
Phys. Lett. B, **689**, 14-17, (2010)

[9] T S Biró, K Urmossy, Z Schram, Thermodynamics of composition rules
J. Phys. G-Nucl. Part. Phys., **37**, 9, (2010)

[10] T.S. Biró, K. Urmossy, Pions and kaons from stringy quark matter.
J. Phys. G. G**36** 064044, (2009)

[11] T. S. Biró, G. Purcsel, K. Urmossy, Non-Extensive Approach to Quark Matter.
Eur. Phys. J. A, **40** 325-340, (2009)

[12] T. S. Biró, K. Urmossy, ALCOR: From quark combinatorics to spectral coalescence.
Eur. Phys. J. ST **155** 1-12, (2008)

[13] T. S. Biró, K. Urmossy, G. G. Barnaföldi, Pion and Kaon Spectra from Distributed Mass Quark Matter.
J. Phys. G. G**35** 044012, (2008)

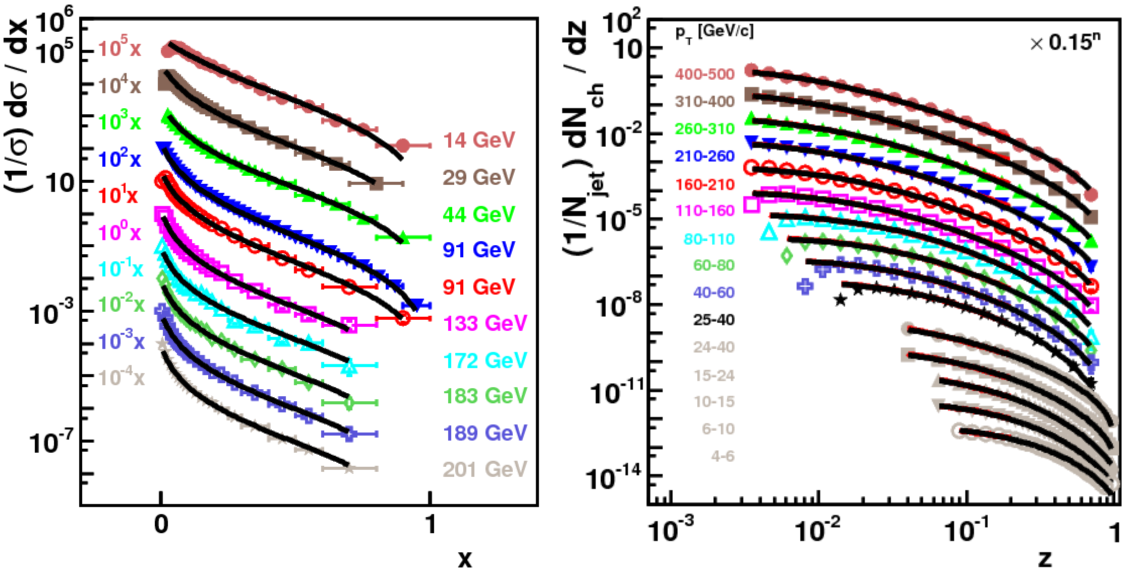

Figure 1: Comparison of the *Microcanonical fragmentation model* to measurements. **Left:** Fragmentation functions measured in e^+e^- annihilations at various collision energies [4]. **Right:** Fragmentation functions measured in pp collisions at $\sqrt{s} = 7$ TeV (ATLAS Collaboration, LHC) [1].

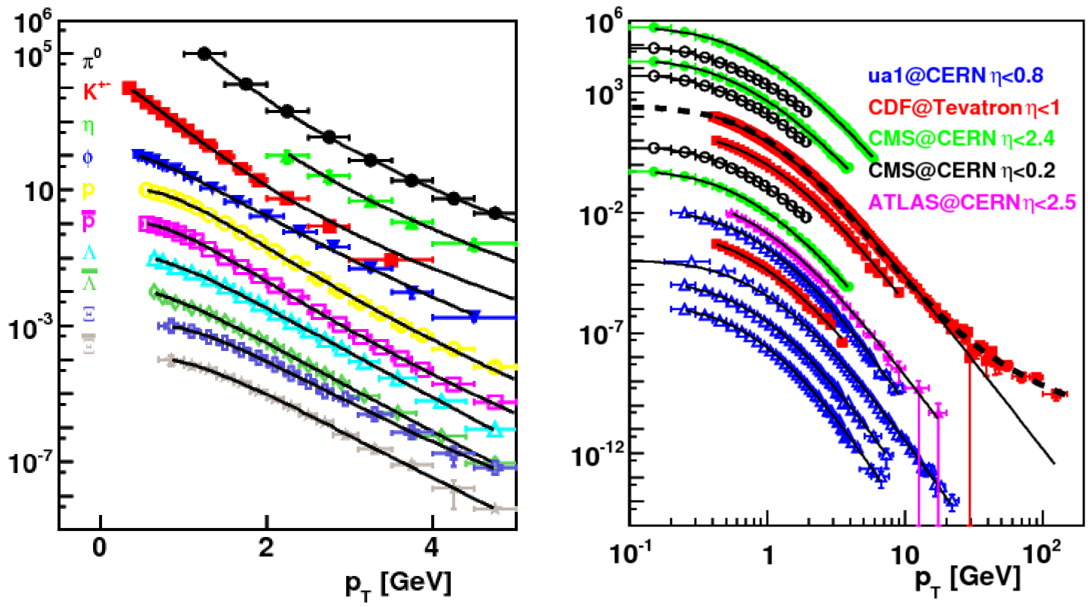

Figure 2: **Left:** Identified hadronspectra measured in AuAu collisions at $\sqrt{s} = 200$ AGeV and results of the *direct quark-coalescence model* based on the Tsallis-Pareto distribution [8]. **Right:** Fits of the Tsallis-Pareto distribution to transverse spectra of charged hadrons stemming from pp collisions at $\sqrt{s} \in [200\text{ GeV}, 7\text{ TeV}]$ collision energies [7].

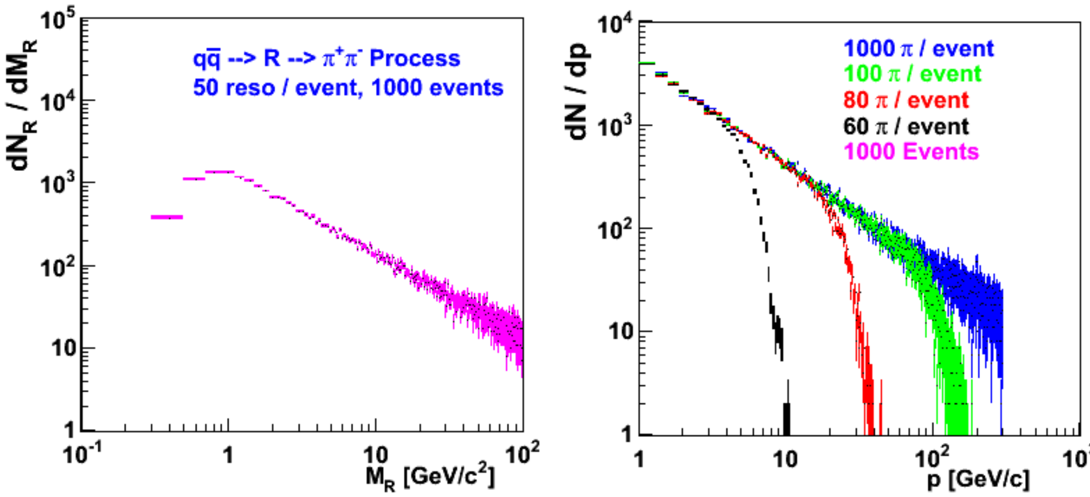

Figure 3: **Left:** Mass distribution of pre-resonances formed inside a non-extensive quark matter. **Right:** Spectra of pions produced in the decay of the resonances for various values of the number of particles produced in each event [5]. The high-energy part of the spectra decreases visibly faster than a power function. This effect is due to the decrease of the phase space induced by the finiteness of the energy of the system.

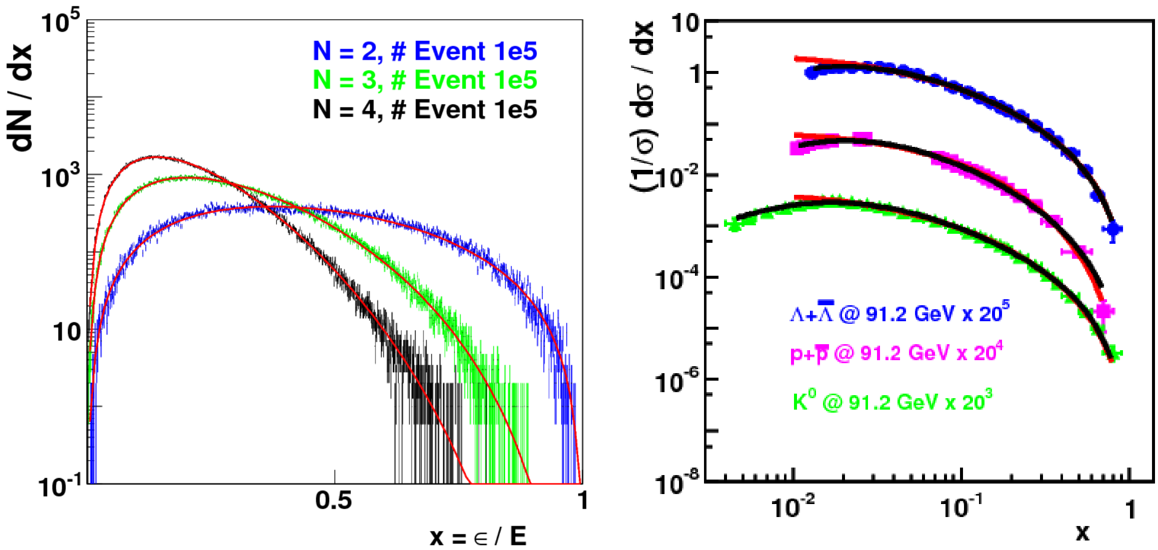

Figure 4: **Left:** Equilibrium single-particle distributions in systems composed of 2, 3 and 4 particles, and described by the energy composition rule above (results of analytic calculations and simulations). **Right:** Analytic results also accounting for the fluctuations of the multiplicity compared to Λ, $\bar{p}$ and K^0 spectra stemming from e^+e^- annihilations measured at $\sqrt{s} = 91$ GeV collision energy [2].