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Dynamics of multi-layer spherical shell systems and its applications in general relativity

PhD Thesis

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Introduction

It is customary to distinguish two types of surface discontinuities within the framework of general relativity. Boundary surfaces, or shock waves, are characterized by a jump discontinuity in the density; and density becomes infinite on surface layers, or infinitesimally thin shells. The latter case is an idealistic limit with finite surface energy density. In the physical reality, it does not exist, but it is a very useful model. For example, this allows us to join exact solutions together. Exact solutions play particularly important role in the general theory of relativity, because it is a highly non-linear four-dimensional theory, therefore, the numerical calculations are computationally extremely extensive. In my thesis, the relativistic dynamics of spherically symmetric shell systems is examined focusing on practical applications.

Relativistic infinitely thin spherical shells play an important role in various dynamical contexts ranging from microscopic to astrophysical systems. For instance, by applying a charged shell as an electron model one may avoid the appearance of negative gravitational mass caused by the concentration of charge at the center. Using families of spherically symmetric thin shells instead of spherically symmetric continuous matter distributions reduces significantly the complexity of evolutionary problems as the dynamics of thin shells may be investigated by using various analogies from the description of the motion of a particle in a one-dimensional effective potential. Accordingly, the quantization of systems comprising thin shells is tractable. Macroscopically stable quark-gluon matter can also be studied with a toy model in which relativistic shells and the MIT bag model are combined. Collapsing dust shells can be used to probe stability or to study energetics of compact objects such as back holes or star models mimicking the properties of black holes. Shells can be used to model matter ejection at certain phases of supernova explosions or in modeling supernova remnants. More realistic radiating shell models can also be constructed and with the help of these models, even the critical collapse may be investigated analytically. As we mentioned before, with the help of infinitesimally thin shells one can construct exact solutions by gluing together spherically symmetric spacetime domains. This way exotic models such as gravastars or wormholes may also be studied. Simple models of large-scale voids in galaxy distributions can also be constructed with the help of shells. The dynamics of spherical shells come into play in some cosmological models, such as higher dimensional brane cosmologies, in which it is assumed that our four-dimensional universe is merely a surface living in a higher dimensional spacetime. Shells play a central role in the bubble inflation model of the early universe. It is widely thought that by studying dynamics of shells, important phenomena such as the focusing singularity at the center or the so-called acoustic singularity can also be studied.

In describing the evolution of families of infinitesimally thin shells, the study of their crossing is essential. Nevertheless, much less has been done in this respect. Moreover, most of the authors do not go beyond deriving the equations of motion for dust shells, or studying only the simplest possible analytic cases. This, in particular, means that almost no results are available for multi-layer shell systems with the generic equation of state. The main aim of the dissertation is to present some new results concerning the dynamics of multi-layer shell systems with the inclusion of of shell crossings. The corresponding dynamical investigations were carried out by using a fast C++ code which made the study of the evolution of systems composed of a large number of shells and with a generic equation of state possible.

After the introduction, in section 2 of my dissertation, some of the basics related to the analytic description of the motion of a single shell are recalled using Schwarzschild and ingoing Eddington–Finkelstein 'time' coordinates. This latter one allows us to evolve shells throughout the entire spacetime, including the black hole region. For comparison, the corresponding Newtonian case is also discussed. Whenever two shells collide the evolution is continued with the assumption that the collision is totally transparent or totally inelastic. In section 3, the equations of the above mentioned type of collisions are derived. In section 4, the time evolution of systems comprising a large number of concentric shells is investigated. The formation of acoustic singularity is studied in detail. The other important outcomes of the investigations is that the phenomena of mass inflation, which is discussed at the end of this section. In the last section, as a special application, the radial stability of the three-layer gravastar model is studied by determining the response to the arrival of a dust shell onto the gravastar surface. The gravastar model is a black hole mimicking object which has received considerable attention because its relation to the concept of dark energy. In this model an interior de Sitter spacetime region is connected to an outer Schwarzschild solution such a way that no event horizon appears.

Aims

Objective of this work was to develop an efficient numerical software for describing newtonian or relativistic time evolution of shell systems with arbitrary equation of state on each shell and allowing collisions. Since previously no more than two shells are studied, new effects are expected to observe, especially in connection with the acoustic singularity and the mass inflation phenomenon. The software was planned to be suitable for using de Sitter interior instead of Schwarzschild vacuum, because this makes possible to study stability of gravastars.

Methods

Most of the results are based on the developed C++ code. The convergence of the numeric algorithm is tested considering collisions. This code has a lot of advantages which are summarized below. However one of the main criteria for the development of the numeric code was the proper theoretical description of multi-layer shell systems.

- The fast numerical calculation is ensured by the use of C++ language. In integrating the equations of motion the fourth-order Runge–Kutta algorithm was applied. Moreover, lots of technical tricks are applied. For example, certain particular results of subroutines are stored in the memory for later usage. This is important, because C++ is an object-oriented language, which means that objects usually protect their inner data.

- The core of the software package is the C++ library containing the basic objects, such as the equation of state, the shell or the shell system. The user can easily generalize or modify objets. Replacing of numerical algorithms can be performed without any problems. A simple variable makes it possible to choose Newtonian, Schwarzschild, or Eddington time for the evolution.

- There are several executable files in the software package for simulating shell systems. Initial data of the system can be stored in a simple text file and edited by hand or generated by scripts. Scripts can be found in the package and usable for thick shell simulations. It is possible to set the resolution of dynamics and the output time step separately. Since visualization of the results is also very important, gnuplot script generators are also available.

The stability studies of gravastars, described in the last section of my dissertation, rely on the examination of the effective radial potential. The examination of the potential is made by a numerical root finding and maximum finding algorithms based on the interval bisection method.

Theses

1. The analytic description of the motion of a single shell is discussed and the required equations are derived in the Appendix. Most of this summary section contains known results, however, lots of important notes and refinements are made. The followings are the most important ones. The appropriate treatment of the sign factors was emphasized. The critical radius is introduced below the horizon, where the sign change can happen. Schwarzschild-shells in equilibrium are described in the parameter space. Carter–Penrose diagrams are clarified for shells. Possible initial conditions of shells are discussed in detail. The mass function is given as a function of radius for linear, broken linear, and polytropic equation of state. The motion is described in the black hole region by using ingoing Eddington–Finkelstein time' coordinates. For comparison, the corresponding Newtonian case is also discussed.

2. The description of a collision of two shells cannot be given without invoking some further assumptions concerning the interaction of the shells. Two extreme cases are discussed. The generic description of the transparent and totally inelastic collisions of spherical shells in spherically symmetric spacetimes is provided in section 3 of my dissertation.

3. In describing the relative motion of multi-layer shell systems a proper synchronization method is introduced. By making use of the developed numerical method for the first time, the relativistic time evolution of numerous shell systems involving large number of thin shells could be made. The formation of acoustic singularities is studied in thick shell mimicking multi-layer systems. In case of uniform initial distributions (radius, mass, velocity, equation of state) the formation of acoustic singularity appears. However, it is found that either type of deviation from the uniform distribution yields a visible dispersion of the shells. The focusing of the shells is lost faster and the spreading is more intensive as the initial distributions get further and further away from uniformity.

Relativistic studies are compared in certain cases to the corresponding Newtonian time evolution.

4. Dynamics of small perturbations in uniform systems are studied separately. There is a tendency for the formation of a crust, represented by the increase of the density of shells, at the edges of widening gaps. Following the widening of a gap, a reversing of the sides also occurs, the innermost shell becomes the outermost and vice versa, as it is clearly visible in the colored figure. Around 'anti-gaps' an increase in the density of shells is also noticeable, although the growing rate is much lower than in case of gaps.

5. The analytic setup is chosen such that the developed code is capable of following the evolution even inside the black hole region. This, in particular, allowed me to investigate the mass inflation phenomenon in the chosen framework. It is also important to note that whenever the collapse and mass inflation occur the blow-up rate behavior is insensitive to the initial data of the shells. I would like to emphasize that beside the numerical investigation of mass inflation, an estimate explaining the main features of the blow-up behavior of the mass parameter of the intermediate region is also provided.

6. As a preparation for the dynamical investigations, the gravastar section begins with a short review of the three-layer gravastar model with distinguished attention to the structure of the pertinent parameter space of gravastars in equilibrium. Then the radial stability of these types of gravastars is studied by determining their response for the totally inelastic collision of their surface layer with a dust shell. It is assumed that the dominant energy condition holds and the speed of sound does not exceed that of the light in the matter of the surface layer. While in the analytic setup the equation of state is kept to be generic, in the numerical investigations three functionally distinct classes of equations of states are applied. In the corresponding particular cases the maximal

mass of the dust shell that may fall onto a gravastar without converting it into a black hole is determined. For those configurations which remain stable the excursion of their radius is assigned. It is found that even the most compact gravastars cannot get beyond the lower limit of the size of conventional stars, provided that the dominant energy condition holds in both cases. It is also shown—independent of any assumption concerning the matter interbridging the internal de Sitter and the external Schwarzschild regions—that the better is a gravastar in mimicking a black hole the easier is to get the system formed by a dust shell and the gravastar beyond the event horizon of the composite system.

Conclusion

The relativistic time evolution of multi-layer spherically symmetric shell systems has been investigated. After recalling the basics of the analytic setup a newly developed numerical code is introduced. This numerical method was made to be capable of following the time evolution of systems comprising of great numbers of colliding shells such that whenever collisions occur they are assumed to be transparent or totally inelastic. The formation of acoustic singularities is analyzed numerically with this apparatus. The chosen analytic setup ensured that the evolution of the considered shell systems can be investigated both in the domain of outer communication and in the black hole region. This made the numerical code capable of studying mass inflation within the thin shell formalism. Beside the numerical investigation of mass inflation, an estimate explaining the main features of the blow-up behavior of the mass parameter of the intermediate region is also provided.

As a special application, the radial stability of gravastars was investigated. Concerning the matter model of the surface of the gravastar three different types of equations of state were applied. In all the investigated cases it is clearly manifested that even a dynamical gravastar cannot be more compact than the smallest possible ordinary stars of the same mass provided that for both types of models the dominant energy condition is guaranteed to hold. This means that the main issue is not that whether gravastars can be distinguished from a black hole. Assuming that they exist in reality it is more appropriate to ask whether they can be distinguished from compact regular stars. It is also shown—independent of any assumption concerning the matter interbridging the internal de Sitter and the external Schwarzschild regions—that the better is a gravastar in mimicking a black hole the easier is to get the system formed by a dust shell and the gravastar beyond the event horizon of the composite system.

Papers containing the results of the thesis

- Merse E. Gáspár, I. Rácz:
  Probing the stability of gravastars by dropping dust shells onto them
  Classical and Quantum Gravity **27**, 185004 (2010) IF: 3.098

- Merse E. Gáspár, I. Rácz:
  On the dynamics of relativistic multi-layer spherical shell systems
  Classical and Quantum Gravity **28**, 085005 (2011) IF: 3.320

Other articles in refereed journals

- B. Kocsis, Merse E. Gáspár, Sz. Márka:
  Detection Rate Estimates of Gravity Waves Emitted during Parabolic Encounters of Stellar Black Holes in Globular Clusters
  The Astrophysical Journal **648**, 411 (2006)

- Merse E. Gáspár, P. Csermely:
  Rigidity and flexibility of biological networks
  Briefings in Functional Genomics, in press (2012)

- T. Hegedus et al:
  Potential application of network descriptions in the conformational changes and protonation states of ABC transporters
  Current Pharmaceutical Design, in press (2012)

Talks

- 5th Workshop of Young Researchers in Astronomy and Astrophysics
  Budapes, 2009

- report to the Hungarian Academy of Sciences
  KFKI RMKI, 2009

- Seminar on Mathematical Relativity
  ESI, Wien, 2011

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- Merse E. Gáspár and B. Kocsis:
  The development of a digital camera with a high dynamic range
- B. Kocsis and Merse E. Gáspár:
  The data analysis for short-term gravitational-wave burst signals with a modified maximum likelihood detection method
- B. Kocsis and Merse E. Gáspár:
  Expectations on the gravitational wave signals associated with cosmic bremsstrahlung events

Virgo Collaboration papers

- *LIGO S6 detector characterization studies*
  Classical and Quantum Gravity **27**, 194010 (2010)
- *Noise from scattered light in Virgo.s second science run data*
  Classical and Quantum Gravity **27**, 194011 (2010)
- *Data quality in gravitational wave bursts and inspiral searches in the second Virgo Science Run*
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- *Very low latency search pipeline for low mass compact binary coalescences in the LIGO S6 and Virgo VSR2 data*
  Classical and Quantum Gravity **27**, 194013 (2010)
- *Tools for noise characterization in Virgo*
  Journal of Physics: Conference Series **243** 012004 (2010)