General theory of the Zitterbewegung

PhD thesis

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by

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Zitterbewegung (ZB) is a fast oscillatory motion of free particles. It has been discovered by Erwin Schrödinger eighty years ago. Present paper deals with mathematical and physical basics of ZB, in special consideration of new phenomena occured in several nanophysical systems, which are similar to Schrödinger's original ZB, but differing from it in some respects.

Main stetements of the paper are the following:

1. ZB is a very general quantum phenomenon, it occurs in lot of quantum systems. Necessary conditions of its occurence are the followings:

- Multi-component quantum system. Its Hilbert space is the tensorial product of a usual Hilbert space of infinite dimension (e.g. space of functions) and of a complex linear space of finite dimension. The physical meaning of the degree of freedom described by the components of the wave function can be very different, e.g. spinor components of Dirac's electron, two atoms on non-equivalent sites in the elementary cell of monolayer graphene, layers of bilayer graphene, valence and conduction bands of solids, two components arising in the process when the Klein-Gordon equation of second order is transformed to system of two differential equations of first order, etc.
- Quasi-free quantum system. It means thet the Hamiltonian governing the dynamics of the system depends only on the operator of momentum (or quasi-momentum), and it does not depend on the position operator. There are two cases: first, the system is really invariant to translations (e.g. relativistic free electron), second, the mathematical desription of the system eliminates the dependence on position (e.g. the band theory of solids which transforms the complicated details of position dependence into the functional form of the multiband dispersion relation). Remark: this condition does not involve that there is not ZB-like complicated motion in the presence of position dependent Hamiltonian, contrary it denotes that there is no definite method to isolate the "smooth" motion corresponding to the classical solution and the oscillating motion superposed on it.
- Coupling of the internal degrees of freedom and the translational degrees of freedom. The most simple form of it is a Hamiltonian containing non-diagonal matrix elements depending on (quasi-) momentum. This relation can be named as "generalized spin-orbit coupling".

In the presence of the conditions discussed above the phenomena of Zitterbewegung occurs in the system under consideration – except of the presence of a special extra symmetry which forbids ZB.

2. The general method which has been published in the paper [C], and which is presented in details in the Thesis is based on the projector decomposition of the operators. It gives the complete description of Zitterbewegung, i.e. it presents the exact time-dependence of the position operator in the Heisenberg's picture. The position operator contains the following terms:

- Operator of the initial position – it is identical to the position operator of Schrödinger's picture, i.e. the gradient operator against the momentum.
- Linear motion of constant velocity. The operator of velocity is a sum of terms, which are tensorial products of projection operators projecting on the different energy eigen-subspaces of the Hamiltonian and of vectors of partial velocities, defined as gradients of the appropriate energy eigenvalues against the momentum. These vectors of partial velocities are generally parallel neither to each other, nor to the vector of the linear momentum.
- A vector of constant shift – this term simply adjustes the initial values of the oscillating terms. Technically it is worth it to melt down into the oscillating terms.
- Oscillating terms, i.e. proper terms describing the phenomenon of the Zitterbewegung. It is shown that contrary to the former general opinion, ZB is not an oscillation of a single definite frequency but all of the "beating" frequencies corresponding to the differences of the energy eigenvalues belonging to a common value of the linear momentum occur. The coefficient matrices of different modes can be constructed using the projectors projecting on the energy eigen-subspaces and their gradients against momentum.

3. Against the common opinion the Zitterbewegung has no direct relation to the special theory of relativity, nor to the phenomena of spin. It is a pure historical accident that ZB occured first time in connection with the relativistic Dirac's equation describing the spin of electron as well. Thus one can understand the occurence of ZB in model systems of solid state and nanophysics, in absence of relativistic velocities: there are multi-component wave functions and generalized SO-coupling in such models, so the appearance of ZB is natural.

4. In what cases is the ZB absent? An interesting example is given: Rashba–Dresselhaus model of solids. Energy eigenvalues depend on the momentum, but the projectors are constant, thus their gradients against momentum are zero, i.e. every ZB coefficient vanishes. Similar situation may be occur in the case of other quantum systems as well.

5. Our general description is applied to the description of models for which ZB was reported in the literature by different authors, using different *ad hoc* calculation methods. The results of all of such papers were recapitulated or corrigated (see our papers [B] and [C]). Moreover we investigated other systems as well, for which the ZB was formerly not discussed in the literature. The description of ZB in the isotropic model of bilayer graphene is first published in our paper [C] and its detailed version [C1]. Similar but more complicated calculations for anisotropic model of bilayer graphene will be shortly published.

Following statements are not discussed in the Thesis, they are party published results of the work performed after the finishing of the Thesis.

6. A new and surprising result is the discovery of the relationship between the ZB and transport phenomena (electric conductivity, Hall effect etc.) of model systems. ZB coefficient matrices occur in the formulae of transport properties: it involves deep connection between these phenomena. Details are published in our papers [A] and [D]. Key step of the calculation is the projector decomposition of Hamiltonian. This relationship can help the (indirect) experimental detection of ZB, moreover it is a promising way to deeper understanding and future controlling of the complicated transport phenomena of systems with non-trivial individual electron motion.

7. Interference between waves of different momentum parameters cause in practice the suppression or fast attenuation of the ZB. Our general theory can help to find the special cases when the special form of the dispersion relation or the momentum dependence of projectors inhibits the decay of ZB, thus it can be a longly surviving mode of motion. Further investigation of the question is under way using the formalism of second quantization.

Publications of the author in topics of the thesis:

[A] József Cserti, András Csordás, and Gyula Dávid *'Role of the trigonal warping on the minimal conductivity of bilayer graphene'*, Phys. Rev. Lett. **99**, 066802 (2007).

[B] József Cserti and Gyula Dávid, *'Unified Description of the Zitterbewegung for Spintronic, Graphene, and Superconducting Systems'*, Phys. Rev. **B 74**, 172305 (2006).

[C] Gyula Dávid and József Cserti, *'General Theory of the Zitterbewegung'*, Phys. Rev. **B 81**, 121417(R) (2010).

[C1] Detailed version of the previous paper: arXiv:0909.2004v3 (2009)

[D] József Cserti and Gyula Dávid, *'Relation between Zitterbewegung and the charge conductivity, Berry curvature and the Chern number of multi band systems'*, Phys. Rev. **B 82**, 201405(R) (2010).