# Ph. D. THESIS

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# Introducing gravitomagnetism to the curriculum on relativity

**Balázs Vető**

**Department of Theoretical Physics**
**Faculty of Science**
**Eötvös University, Budapest**

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**Introduction**

The phenomenon of gravitomagnetism means interaction between moving masses in the Newtonian mechanics. This interaction is undetectable in terrestrial laboratories because of its weak effect. The gravitomagnetic interaction became known as the low velocities and weak fields approximation of general relativity in the middle of the 20-th century. Its experimental demonstration was carried out in the Gravity Probe B experiment ended in 2007. There is a lack of gravitomagnetism in the teaching of physics in higher education in Hungary.

**Objectives**

In this thesis, I am introducing a methodology for adding gravitomagnetism to the university course on relativity. The target audience is physics master's degree students and last year undergraduates interested in the topic. The topic of gravitomagnetism could be a stand alone course or could be taught as part of a course focused on relativity. The content of the dissertation could give the framework of a seminar series on gravitomagnetism and could also be used as lecture notes. A selection related to relativity can also be made to only use parts of the dissertation.

The design of the curriculum is aimed at giving topical knowledge to students as well as helping the student's ability to intuitively think about the subject. The curriculum is based on real life applications of gravitomagnetism. The methodology is built on the already acquired mathematical toolbox of electromagnetism. Using this earlier acquired knowledge of electromagnetism, the students can explore a new subject and improve their intuition. The curriculum provides subject specific knowledge and uses a simpler mathematical formalism than research papers.

**Results and discussion**

1. *Electromagnetism and special relativity*
   In the 19th century, the Coulomb interaction between charged particles and the electromagnetic interaction between moving particles were treated as two independent phenomena, even though they were both referred to as electromagnetism. In the dissertation, I show that when using special relativity, the existence of the electromagnetic force between moving particles is the consequence of the laws of electrostatics and special relativity.
   This derivation changes the way one thinks about the subject as it shows the causality relationship between two already known phenomena.
2. *Gravitomagnetic phenomenon*
   Textbooks on gravity do not usually mention that between moving masses there is another interaction in addition to gravity – called gravitomagnetism. In this section, I show a methodology to explain the relativistic force occurring between

moving masses. I illustrate that the existence of gravitomagnetic phenomenon is a consequence of Newtonian gravity and special relativity.

Since special relativity is not able to give an exact, quantitative description of gravity, in the second part of this point, I show a quantitatively better approximation of gravitomagnetism based on general relativity, in the case of low fields and low velocities. The magnitude of the Lorentz force derived from special relativity is a factor of two less than shown by experimental evidence, but its methodological significance is great because it can show the existence of the phenomenon without using general relativity's much more complicated formalisms.

3. *Gravitomagnetic laws derived from general relativity*
   I summarize and describe the laws and relationships of gravitomagnetim in SI units and compare them to the laws of electromagnetism.
   The methodological value is coming from the fact that learning from analogies between gravitomagnetism and electromagnetism, makes the new laws more palatable for the students.
4. *Applications of gravitomagnetism*
   I show the practical applications of gravitomagnetism based ont he following examples:
   - Gravitomagnetic field of moving masses
   - Gravitomagnetic force and torque
   - Gravitomagnetic induction
   - Gravitomagnetic dipole moment of rotating bodies
   - Gyroscope in a gravitomagnetic field
   - Gravitomagnetic effect of remote bodies on rotating Earth
   - Gravitational waves

   The applications and thought experiments give a deeper understanding of the topic and can be worked though by the students on an individual basis.
5. *Gravitomagnetic phenomena related to the Earth*
   By solving the gravitational Poisson equation, I determine the gravitomagnetic field of the rotating Earth and demonstrate that it is a dipole field in any point which is beyond the Earth's surface for an external observer. Using the gravitomagnetic Biot-Savart law known from electrodynamics, I derive the magnitude of the gravitomagnetic field of distant objects in the frame of the rotating Earth. I point the relation between gravitomagnetic field of the mass of the Universe and Coriolis force. The methodological value of the derivation is the application of knowledge in a new situation.
6. *Understanding the Gravity Probe-B experiment using gravitomagnetism*
   I deduce the gravitomagnetic field as seen by an observer on a satellite orbiting the Earth. In this case, the field has two components: the field induced by Earth's rotation and the field induced by Earth's virtual motion around the satellite. I show the precession angular velocity of a freely rotating gyroscope placed on a

satellite orbiting the Earth on a polar orbit using gravitomagnetism. The result is in agreement with the Schiff law derived from general relativity as well as the results of the Gravity Probe-B experiment. Using my work, the students can move onto the understanding of geodetic precession and the Lense-Thirring effect without having to work through general relativity.

7. *Electromagnetic analogies of geodetic precession and the Lense-Thirring effect*
In order to further illustrate the analogy between electromagnetism and gravitomagnetism, I present a thought experiment to examine the Coulomb force between two insulating bodies with opposite charge. One of the bodies is a gyroscope with a mass much lower than the other body. If this gyroscope is orbiting around the other body with a much larger mass, than its spin axis will precess similarly to the gyroscope in the Gravity Probe-B experiment. In that case, a phenomenon analogous to geodetic precession will appear, with an extent depending on the specific charge of the two bodies.
If the central, heavier body is also rotating around its own axis, than the electromagnetic effect analogous to the Lense-Thirring effect will also appear. As a consequence, the orbiting gyroscope's angular momentum vector will be displaced towards the direction of the rotational direction of the central body.
The methodological value of the presented thought experiment is reversing the analogy, and using it in the other direction to arrive to new physical consequences. In the earlier examples, one explores gravitational consequences of known electromagnetic phenomena. In this one, however, one is arriving to electromagnetic consequences of gravitomagnetic phenomena. That way of thinking proves that existing mathematical and physical knowledge can be successfully applied in new fields of physics.

## Publications supporting the Thesis

Vető B. Az elektromos kölcsönhatás a speciális relativitáselmélet szemszögéből *Fizikai Szemle* 2009/4. 127-131.

Vető B. Gravitáció és gravitomágnesség, *Fizikai Szemle* 2010/9. 296-299.

B. Vető Gravity Probe B experiment and gravitomagnetism, *Eur. J. Phys.* 2010. vol. 31. p. 1123-1130.