MTA KFKI Research Institute for Particle and Nuclear Physics
Department of Theoretical Physics

Localised solutions and their perturbations in field theory

PhD THESIS

Written by: Árpád László Lukács

Supervisor: Prof. Dr. Péter Forgács

MTA KFKI Institute for Particle and Nuclear Physics

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Roland Eötvös University
Faculty of Science

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Roland Eötvös University, Physics PhD School

Head of PhD School: Professor Dr. Zalán Horváth

PhD Programme: Particle Physics and Astronomy

Head of PhD Programme: Professor Dr. Ferenc Csikor

Supervisor: Professor Dr. Péter Forgács

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1 Introduction

According to our present understanding, the Universe formed in a hot and dense state, and immediately started to expand and cool. In present theories of Particle Physics , spontaneous symmetry breakdown plays a major role. Symmetries that are present at high energy are broken at low energy. As a consequence of these two, the Universe in its early state must have gone through a series of phase transitions.

The braking of Grand Unification symmetry takes place at an energy scale of 10^{15} GeV, and electroweak symmetry is broken at approximately 250 GeV, therefore phase transitions had to occur at 10^{-36} and 10^{-12} seconds after the Big Bang. Inflation took place in between, from 10^{-36} to 10^{-32} seconds.

Among theories exhibiting spontaneous symmetry breaking, many have solutions with a well defined spatial expansion. During phase transitions, objects corresponding to these, called defects, form almost unavoidably. These include strings, monopoles and domain walls. Out of these, probably cosmic strings have the most important cosmological consequences. After the defect formation, the network of cosmic strings evolves dynamically. The properties of the network of cosmic strings might play an important role in generating the fluctuations of the cosmic microwave background radiation and in structure formation (i.e. the production of the initial density fluctuations that later lead to the formation of galaxies, clusters and the large–scale structure of the Universe (voids and filaments).

The fluctuations of the cosmic microwave background have recently been studied in high detail. The best agreement between theory and experiment is reached when the contribution of cosmic strings is taken into account besides contributions of other sources. The best fit is achieved by setting the string tension to $G\mu = (2.04 \pm 0.13) \times 10^{-6}$ (corresponding to approximately GUT scale strings) and the ratio of string contribution (taken at the 10^{10} multipole component) is then $f_{10} = 0.11 \pm 0.05$ (Hindmarsh, 2008).

The existence of cosmic strings is predicted by a variety of Grand Unified Theories, and Superstring Theory also allows configurations similar to cosmic strings (cosmic superstrings and D–strings). Similarly, certain supersymmetric theories and and field theoretic models of dark matter also possess string solutions. On much lower energy scales than GUTs, string solutions exist also in the electroweak sector of the Standard Model (however, all known ones turned out to be unstable).

Objects, whose mathematical form is highly similar to that of cosmic strings also play an important role in Condensed Matter Physics. There is a close analogy between cosmic strings and vortex lines of superfluids and Abrikosov vortices in Type II superconductors. These analogies take on added importance due to fact that with their help, predictions of theories applied in Cosmology, can be tested in laboratory experiments.

The extended objects examined in the present thesis are described by classical field theory solutions. Despite this fact, they retain their importance in Quantum Theory as well, as field fluctuations about a classical solution can be quantised as well as fluctuations about the vacuum, and thus a classical localised solution can assume the role of the zeroth order of a semiclassical approximation. On the other hand, in the case of the solutions important in Cosmology, the classical description is sufficient in itself.

The simplest, one dimensional localised solution, the kink interpolates between discrete degenerate vacua. This object is known to exhibit a particle-like behaviour, in the sense that the dynamics of its center of mass follows Newton's equation of motion in a good approximation. A problem of high importance, having ample literature, is the motion of vortex lines in non-relativistic hydrodynamics. In the two dimensional case (which can be applied to e.g. a system of vortex lines in a rotating superfluid) they can be described with the help of a simple equation of motion. The momentum of the vortices, however, depends on the positions of the vortices and not the time derivatives thereof, and thus the dynamics of the vortices is first order in time.

2 Aims

The aims of my work were to examine the following problems concerning the dynamics of extended field theoretical solutions:

1. A description of the kink–radiation interaction in one dimensional models. The primary goals were obtaining the leading order scattered waveforms and the description of the motion of the kink interacting with radiation.

2. A solution of the scattering problem of global vortices in relativistic scalar theory. I intended to perform the calculations in the simplest relativistic theory admitting vortex solutions, namely the Goldstone model with global $U(1)$ symmetry. Primary aim was the calculation of the force acting on the vortices due to radiation.

3. Examination of the stability of twisted strings in the $SU(2)$ symmetric extended Abelian Higgs model. This theory can be obtained as the semilocal limit of standard electroweak theory.

4. In most Particle Physics models exhibiting spontaneous symmetry breakdown, the symmetries of the theory are broken by a multicomponent scalar field. Thus, from an analog

model point of view, it is of high importance to examine the similarities and differences between string solutions of these and those of Ginzburg–Landau theories of Condensed Matter Physics. In the latter, however, the potential does not have the fundamental symmetries of Particle Physics models. My aim here was therefore to obtain the most general two component extension of the Abelian Higgs model which admits twisted strings with cylindrical symmetry, and find its string solutions.

3 Methods

The above problems fall into the realm of classical field theory. In the case of localised solutions however, nonlinearities play a fundamental role, therefore, an exact solution of the field equations is unlikely. Thus, in the process of solving the proposed problems, I applied approximation

In the case of the kink – radiation system, I have expanded the field equations about the kink, in a power series in the amplitude of the incoming wave. In the first few orders, I have shown that the motion of kink follows Newtons's equation. Thus, neglecting deformation effects, the problem reduces to the calculation of the force acting on the kink. This was done by enclosing the kink in a box, and evaluating the momentum balance at the boundary of the box.

In the case of the vortex–radiation system, neither the background vortex, nor the scattered wave can be obtained analytically. With the help of a suitable Ansatz, both problems can be reduces to the solution of the respective radial equations. In this way, I have solved boundary value problems of ordinary differential equations, with the shooting to a fitting point method and in certain cases collocation (using the COLSYS package by Ascher). The scattered wave was obtained in a partial wave sum, and the force acting on the vortex was evaluated by substituting the asymptotic fields in the stress–energy tensor and thus calculating the momentum balance in the wave zone.

In the solution of the instability problem of the twisted strings, I have also applied the partial wave expansion of perturbation functions. The radial functions of the partial wave components were again solved using the shooting method. Apart from the numerical solution, I have developed a semi–analytical approximation of twisted strings close to the bifurcation, based on the multiple scale method. With the help of that, I have shown that the perturbation problem of twisted strings is a deformation of that of the embedded ANO–strings, to the description of which quantum mechanical perturbation theory can be applied.

I have obtained the vortex solutions in non $SU(2)$ symmetric two component Abelian Higgs models with the aforementioned numerical methods.

4 Theses

1. I have obtained a perturbative solution to the problem of a kink interacting with radiation. The first order radiation passes through the kink without reflection in both the ϕ^4 and the sine–Gordon model. Accordingly, the well known radiation pressure, quadratic in the amplitude of the incoming radiation, vanishes.

   Due to nonlinearities of the models, higher order waves emerge from the kink. In the case of the Goldstone–model, the fourth order force contribution due to these waves acts in the direction of the source of the radiation, i.e. the radiation pressure is negative. This can be explained by the excess momentum appearing behind the kink due to the higher order waves. In the case of the sine–Gordon equation, the force is zero in all orders.

2. I have shown that the effect is stable against small perturbations of the ϕ^4 model. Although in the general case, even the smallest perturbation of the potential leads to the appearance of ordinary quadratic radiation pressure, if the amplitude of the incoming wave if larger than a certain critical value, fourth order negative radiation pressure can dominate. The importance of this “structural stability” lies in the fact, that in most cases, these field equations are used as effective models, and deviations from the models are unavoidable.

3. I have examined the scattering of waves off the vortex in the relativistic Goldstone–model. The leading order force component is quadratic in the amplitude of incoming waves in this case. However, when the massive wave mode of the model is scattered, the force again acts in the direction of the source. Unlike in the case of the kink, the force here is quadratic, the excess moment behind the vortex is not caused by nonlinearly scattered waves, but by scattering of the massive wave into the Goldstone mode of the model.

4. With the linearisation of the field equations I have shown that twisted strings in the $SU(2)$ symmetric extended Abelian Higgs model are unstable. They possess growing modes, corresponding to negative eigenvalues of the perturbation operator. These modes can be parametrised with their wavenumber along the string, $k \in [-k_{\rm m}, k_{\rm m}]$. The maximal wavenumber, $k_{\rm m}$ is a function of both the β parameter of the theory and the twist ω of the string.

   Twisted strings bifurcate with embedded ANO ones at a certain value of the twist, $\omega = \omega_{\rm b}$, where $\omega_{\rm b} = \omega_{\rm b}(\beta)$. The energy of twisted strings is below that of the embedded ones. In the $\beta \to \infty$ limit, $\omega_\mathrm{b} \to \infty$. Thus, this limit can be used to study vortices far from the bifurcation, which is not feasible numerically otherwise. In the $\beta \to \infty$ limit, I have obtained results similar to ordinary cases.

5. I have obtained a semianalitical description of the bifurcation of twisted strings and embedded Abrikosov–Nielsen–Olesen ones at $\omega = \omega_\mathrm{b}$, based on a multiple scale approximation. Using this description of the bifurcation, I have shown that the perturbation problem of twisted strings is a deformation of that of the embedded ones. Based on this result, I have given a solution to the instability problem, in the form of a perturbation series expansion, with the instability problem of embedded strings as the zeroth order. This was applied to a further examination of the nature of the instability. I have determined that close to the bifurcation, strings can lower their energy by expansion. Further away from the bifurcation, a local formation of a lump on the string still lowers its energy. In accord, it seems probable that twisted strings initially expand similarly to embedded ones. Long time dynamics and the determination of the asymptotic state however require further studies. In these, radiation effects should play a major role.

6. I have obtained the most general potential for a two component extended Abelian Higgs model, that admits cylindrically symmetric string solutions. I have found that in this model, two patterns of spontaneous symmetry breakdown are possible. In one of them (1VEV case), one of the field components obtains nonzero vacuum expectation value, while in the other (2VEV), both components do. I have found that twisted string solutions exist in the 1VEV case, while in the 2VEV one, solutions similar to ANO–strings exist. In the first case, twisted strings have lower energy than embedded ANO ones, and for general potentials, they possess a probably stable, lowest energy limit, which exists simultaneously with embedded ANO strings, with significantly lower energy.

5 Conclusion

The interaction of extended solutions and radiation in field theory exhibits interesting features. One of these effects is what we have called "negative radiation pressure", in which, unlike in the case of ordinary radiation pressure, the force acting on a localised solution points in the direction of the radiation source.

This effect, although with slightly different mechanisms, is exhibited in both the case of ϕ^4 kinks and global vortices. Of the two, the mechanism of the latter one is probably more general, since in this case, the linear scattering is not necessarily reflectionless. It suffices if the theory admits both massive and massless modes, and a massive mode is scattered into a massless one sufficiently strongly at low angles.

I have established that twisted semilocal strings possess unstable modes. These modes are members of a one-parameter family, parametrised by their wavenumber along the string, $k \in [-k_{\rm m}, k_{\rm m}]$. The maximal wavenumber $k_{\rm m}$ is a function of both the parameter β (the squared ratio of the scalar and vector mass) of the theory, and the twist ω of the string.

In the non $SU(2)$ symmetric two component extended Abelian Higgs model, the physics of strings is even more rich. I have shown that in this theory, often both embedded ANO–strings, and a zero twist limit of twisted strings exist simultaneously. The latter one, having significantly lower energy than embedded strings, are probably stable.

6 Publications

6.1 Papers containing the results of the thesis

1. Forgács, Lukács és Romańczukiewicz: *Negative radiation pressure exerted on kinks*, Phys. Rev. **D77** 125012 (2008)

2. P. Forgács and Á. Lukács: *Instabilities of Twisted Strings*, JHEP **0912**:064 (2009)

3. P. Forgács and Á. Lukács: *Vortices in the non $SU(2)$ symmetric two component Abelian Higgs model*, in preparation

6.2 Other scientific publications

1. Z. Perjés and Á. Lukács: Canonical Quantization and Black Hole Perturbations **In** *Fundamental Interactions and twistor-like methods - XIXth Max Born Symposium* (Jerzy Lukierski and Dmitri Sorokin eds., non refereed), AIP Conference Proceedings, 2005

2. V. Czinner, M. Vasúth, Á. Lukács and Z. Perjés: *Covariant Linear Perturbations in a Concordance Model*, International Journal of Modern Physics **A20**, 5671 (2005)

3. V. Czinner, M. Vasúth and Á. Lukács: *An Analytic Approach to the Late ISW Effect in a Λ Dominated Universe*, International Journal of Modern Physics **A20**, p.7233 (2005)

4. Gy. Fodor, P. Forgács, Z. Horváth and Á. Lukács: *Small amplitude quasi-breathers and oscillons*, Phys. Rev. **D78** 025003 (2008)

6.3 Popular science

1. Lukács: *Evaporating black holes in the laboratory* (in Hungarian), TéTéKás Nyúz, **XXXIII** 17 (2006).

6.4 Talks

1. *Quasi–normal modes* (in Hungarian), on the ELFT Summer School "Measurements and Einstein's Theory of Gravitation", 28 Aug - 1 Sep 2006, Gyöngyöstarján

2. *Negative radiation pressure in nonlinear field theories* (in Hungarian), KFKI EFFO Seminar, 26 Feb 2007

3. *Can the radiation pressure acting on the kink be negative?* (in Hungarian), ELTE Particle Physics Seminar, 25 Apr 2007

4. *Examination of the stability of twisted vortices* (in Hungarian), ELTE Particle Physics Seminar, 28 May 2008

5. *Vortex – radiation interactions in ϕ^2 theory*, Strong interactions in QFT, Fürstenfeld, Austria. 15–17 Apr 2009

6. *Twisted vortices without $SU(2)$*, 4^{th} Austrian-Croatian-Hungarian Meeting, Quantum Fields and Quark Matter, Rab, Croatia, 30 Aug – 4 Sep 2009