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Chaotic phenomena in celestial mechanincs

short summary of the PhD results

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Introduction

Computations in celestial mechanics kept the belief in astronomers during many centuries that motions of the planets can be predicted arbitrarily long time to future. However, in the second part of 19th century appeared many problems that showed the trigonometrical series describing the celestial movements are not convergent in any cases, especially close to the resonances. Beside these things Poincaré showed at the turning of the 20 century that the gravitational three-body problem does not have an anlytical solution. His investigations also contain that there are various types of motion depending on initial conditions in above mentioned system. In 1950-60s there was born a precise mathematical explanation about chaotic behaviour in conservative systems (KAM-theory). Not much latter, the information technology became enough advanced to compute the motions very long time and to study the phase portraits that contains large number of initial conditions.

It had been clear that there is no required to study complicated and high dimensional systems because chaotic phenomenon could also evolve even in three dimensional phase space. The differential equations describing the system contain the chaoticity essentially but this quality emerges only by numerical investigations. Usually, the nonlinear behaviour means that the phase space is a 3 dimensional flow at least, therefore, we can point out that the chaotic behaviour is not the exceptional but the typical in nature.

The chaotic motions in frictionless systems gave a new aspect of the behaviour of them in many subject of physics. Celestial mechanics did not dropped out from development. The new concepts, methods, and outlook used in nonlinear dynamics infiltrated also to astronomy. In last decades the main topic is the investigation of chaos in the Solar System and the stability investigation of planetary systems around distant stars.

In my thesis I give a detailed numerical investigation the Sitnikov problem. This system is a particular case of the restricted three-body problem. The Sitnikov problem is a periodically dumped system, and therefore, time dependent. Consequently, one can obtain a wide range of chaotic behaviour in it, although, the motion of the third massless body is only one dimensional. The structure of the three dimensional phase space of the Sitnikov problem was not investigated up to now. In my work I have mapped the whole phase space numerically, the structure of the phase portraits and their relation with initial positions of the primaries has been also explained. The investigations showed an interesting behaviour in the phase space that can not understand with classical chaotic phenomenon (i.e. permanent chaos, chaotic bands). Namely, there are initial condition far from regular domains in the phase space that are origins of long lived motions belonging the system.

In recent years the transient phenomena were brought into focus in field of research of dynamical systems. This feature appears as chaotic scattering in conservative systems, to study this phenomenon has seemed a pioneer work in celestial mechanics. The simplicity of the Sitnikov problem allowed me to show that the finite time chaotic motions are also very important in planetary motion. In my thesis I showed that there are initial conditions related to the motions which remains finite time in the system and during this term there are chaotic. Quantitative investigations result in the instability of transient chaos is much grater than permanent chaos. Finaly, It is also shown in this work that the sticky orbits located at the margin of the regular domains are linked with transient effects.

Aim of my work

The main aspect was in my study to show the chaotic behaviour via simple dynamical system. It is important to note here, the thesis build up in that order as I came to know the problem. After look over the literature and many consultations with my collegues, it was clear that there are numerous details that are not familiar. One of my ideas was to show a complete picture about the 3 dimensional phase space of the Sitnikov problem. The application of concepts and methods of the transient chaos in celestial mechanics was an other aim of mine. To show that the well-known chaotic saddle also exist in dynamical astronomy and it is responsible to the long lived chaotic transients in celestial motions as well. The study of finite time chaotic behaviour could bring a new way to investigation of exoplanetary systems.

Applied methods

The more suitable method to solve nonlinear differential equations is the numerical integration. I have developed C-codes to get the solutions of the equations. The integrator was a 4-order Runge–Kutta method. Chaotic dynamics shows ordered structures in suitable choosed slice of the phase space. These slices one can obtain with known processes like the stroboscopic map or Poincaré Surfaces of Sections (the latter is not used in the computations because the nature of the problem). To investigate the straggling points in the phase portraits, escape times have been introduced. [1] The method gives

[1] Method of escape times in the Sitnikov problem was published by Rudolf Dvorak in 2006 [i]. Dvorak's results are in very good agreement with Figure 3.14 in my thesis where Lyapunov characterestic exponents were used.

how long the third massless body remains close to the binaries. It means, when the testparticle leaves the system the actual integration time will be stored. The results are in good agreement with the contour maps of Lyapunov exponents the other method used in the work. The escape times already were close to the concept the escape rate mentioned in literature. After this, it was very easy to get the average lifetime which characterize the finite time chaotic behaviour in a dynamical system. A scattering function was also taken in order to study the problem in scattering point of view. This function reveals wether transient chaos appears in the system. I have choosen a simple numerical method to design the chaotic saddle [ii]. The quantitative properties and the fractal dimension of the chaotic set have been determined with familiar methodes from textbooks.

Results

1. I have showed the relation between the position of the primaries and the periodical structure of the three dimensional phase space. The tori and the stable periodic orbits sitting inside of them are rotating with defined period round the central equilibrium point. The secondary resonancies and the invariant curves around them follow the same behaviour. It has been shown that the scattered points between the regular islands are the images of the escaping trajectories. The rotational time of these disordered trajectories is varying, therefore, they can get through the invariant tubes and leave for far away in phase space. [1]

2. The numerical investigations of the escape times confirm that there are domains in the phase space away from the central regions where one can observe long lived motions. These trajectories trace out a fractal set in the phase portraits. [1]

3. The scattering function $\Theta(z_0)$ has been defined in the phase space of the Sitnikov problem. It can help to find regions where initial condtions responding to finite time chaotic behaviour are located. I have compared the shape of the scattering function with structure of the phase portraits. The results showed clear, $\Theta(z_0)$ describes the dynamics of the system well and it is a good starting point to study the transient effects. [2]

4. For a given parameter ($e = 0.57$) the escape rate ($\kappa \approx 0.1268$) and the average lifetime of chaos ($\tau \approx 7.88$) has been determined.

In order to show that the escape rate is an invariant quantity I have choosen initial conditions from different domains in phase space. The exponential decreasing was the same in any case. Consequently, κ is a constant for given parameter. [2]

5. When I had the right initial conditions, I have drafted the chaotic saddle numericaly. The pictures showed that the structure of the chaotic set is a multiple fractal. This fractal set contains also a stable and an unstable manifold. They give answer to the filaments in contour map of escape times and also make the trajectories originating from the wall of the tori clear. [2]

6. The chaotic saddle also has been studied quantitatively. The fractal dimension $D = 1.812$ and the Lyapunov exponents ($\lambda = 1.34$) were obtained. Comparing the Lyapunov exponents (exponents from chaotic band againts exponent of the saddle) one can see that the transient chaos is more robust (more unstable) than the permanent chaos. This result support an other phenomenon, namely, the range of the chaotic saddle is much grater than the size of the chaotic bands. [2]

7. Investigations of the phase space show the relationship between the stickiness effect and the transient phenomenon. Sticky orbits appear like permanent chaos in phase space because the chaotic saddle become a two dimensional fractal close to the border of the regular regime. The change of structure of the chaotic invariant set is a consequence of the power law decay of the non-escaping trajectories. The exponent of this power law is $\sigma = 1.03$. [2]

Conclusions

Although the Sitnikov problem is a beloved dynamical system and there are many papers deal with it, the three dimensional phase space structure gives a new insight to the problem. Another important phenomenon is that escape times allow us to obtain the stable manifold of the chaotic saddle in the phase space. Te results plotted on contour map raise many questions about the dynamics and structures in pase space. The answers I have found in transient behaviour. After the results of my PhD thesis, I can summarize my conclusions in two thought.

First, we have seen the finite time chaos is more extended in phase space than the permanent chaos. Therefore, if we study a conservative system, an extended part of phase space would be investigated, for instance far from regular islands or chaotic bands. Quantitative results show that the chaotic saddle is more unsable than the chaotic bands between the tori. It means the motions evolving close to the chaotic invariant set are more sensitive to the initial conditions than the trajectories in chaotic bands. These statements warn us that we have to take into account the influence of the transient chaos to the dynamics even in a simple system in celestial mechanics.

Furthermore, we have seen that the sticky orbits (i.e. trajectories that come close to the wall of the tori and spend long time there before they leave the system) are in close relationship with transient phenomena. One can ask wether it is such an initial condition originating of a trajectory that spend infinite time at the torus. Based my results I suppose, yes. But the chance to find such a trajectory is very small because it requires to hit an exact point of the chaotic saddle. And it is well known the saddle is a zero measure set. So, these results show that the Kantz-Grassberger relation (link between the permanent and transient chaos) holds in conservative systems and also in celestial mechanics. In other words, if a trajectory spend long time around a torus, it would "walk" densely a given part of the phase space. But if it makes to hit exactly a critical initial condition, namely a point of the saddle, then the trajectory going to remain close to the torus and tracing out a chaotic band which is not distinguishable from the permanent chaos anymore.

Publications in subject of the thesis

Papers give the backbone of the thesis

[1] Kovács, T.; Érdi, B.:"The structure of the extended phase space of the Sitnikov problem" *Astronomische Nachrichten*, **328**, No. 8, 801-804. (2007)

[2] Kovács, T.; Érdi, B.: "Transient chaos in the Sitnikov problem" *Celestial Mehanics and Dynamical Astronomy* under publishing

Other papers in subject of the thesis

- Kovács, T.: "The 1:2 resonance in the Sitnikov problem" *In: Proceedings of the 4th Workshop of Young Researchers in Astronomy & Astrophysics; Publications of the Astronomy Department of the Eötvös University (PADEU)*, Edited by E. Forgács-Dajka, 2006, Vol. **17**. on-line version

- Kovács, T.: ”Chaos in simple dynamical systems” *In: Proceedings of the 4th Austrian Hungarian Workshop for Young Researchers On Celestial Mechanics Publications of the Astronomy Department of the Eötvös University (PADEU)*, Edited by Á., Süli, F. Freistetter and A. Pál, 2006, Vol. **18**. p.165-173

Referencies

[i] Dvorak, R.:”The Sitnikov problem – a complete picture of phase space” In F. Szenkovits and B. Érdi, editors, *PADEU*, **19**, p 129. Babes–Bolyai University, Cluj University Press, Cluj Napoca, Romania, (2006)

[ii] Tél, T.; Gruiz, M.: ”Chaotic dynamics” Cambridge University Press, Cambridge, UK, first edition, (2006)