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Traffic Modeling of Communication Networks

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THESES OF PHD DISSERTATION

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1 Introduction

The problem of statistical description and modeling of communication networks based on circuit switched technology was solved in the beginning of the 20th century. The same issue popped up in the 1970s when computer networks based on packet switched technology and the Internet started to develop. With the increasing size of the network and complexity of the generated traffic more and more phenomena were observed that could not be explained by the earlier models. Such phenomena are for example that the traffic rate measured at a certain point of the network (i.e. the number of bytes traversing the link in a time unit) shows self-similarity and long-range dependence in time, there are large fluctuations in the length of the active periods of the routers and the power spectrum of the round-trip time on a certain path decays according to power law. These phenomena are general in the sense that they also occur in physical and biological systems as well as in social and financial systems thus, methods of statistical physics can be applied in their analysis.

The routers in the network are able to forward incoming packets at a certain rate. If the rate of the incoming packets is larger than the outgoing rate then they are stored temporarily in a buffer and will be served later. If this overload situation is persistent then large delay and packet loss occurs. Due to the self-similar nature and high variability of the traffic rate the most challenging task in the design and engineering of the network elements is to compute the statistics of the queue length. Moreover, the traditional models usually underestimate the queue length experienced in real networks.

This calls for the construction of models that considering the basic mechanism of the protocols (Transmission Control Protocol/Internet Protocol, TCP/IP), account for the large fluctuations in the traffic i.e. usually long idle parts are followed by many consecutive packets close to each other (burst arrivals), and the heavy-tailed distributions and at the same time provides a tractable method for the analysis of the system behaviour.

Although applying Poisson and Erlang statistics in traffic modeling of computer networks has its limitations, a more general framework, the matrix analytic methods can provide feasible and well-fitting models on a certain range of

timescales. The purpose of the dissertation is the analysis and modeling of traffic behaviour in computer networks. The methods are based on generalized Markov models that describe heavy tails for several scales. The first two theses deals with the dynamical properties of TCP. The third and fourth theses show an application of the matrix analytic functions through a queuing problem and provide an optimisation method for the matrix representations.

2 Research objectives

One of the most important research tasks was to understand the processes determining the behaviour of network traffic and to find those protocols and rules contributing to the phenomena experienced in packet level measurements. Another task was to set up models based on the basic mechanisms of the network elements to compute the most important parameters describing the traffic.

The applicability of the Markovian approach used in the model is an important issue in telecommunication. The theory of matrix analytic methods provides a framework that can be used to model the packet level processes over a wide range of scales. In infinite-server queuing systems the queue length statistics generally can not be computed exactly. The question arose if restricting the case to the matrix analytic functions (Markov arrival processes and phase-type distributions) the problem can be solved or not. The matrix analytic methods can be applied directly to teletraffic scenarios where the states and state transitions of the underlying Markov-chain correspond to certain events in communication. However, the number of states can easily reach a value so that handling the problem numerically will not be feasible. Therefore I wanted to use a theoretical approach to reduce the number of states (i.e. the size of representation) that clarifies how large the representation should be for a given function and how complex its application can be.

3 Used methods

During my work I used the concepts and basic theorems of stochastic models, specifically Markov-chains and the theory of queuing systems was applied as well. Based on the knowledge of the basic mechanisms of the TCP/IP protocol and the different elements using it I applied these mathematical theories to the investigations of random processes in communication networks.

The theory of matrix analytic methods is an extension of Markovian models that is able to account for a broader class of the processes. The distribution functions can be represented by embedded Markov-chains, matrices and they can be mapped to a finite dimensional vector space where the representation matrices correspond to polytopes. During the analysis I used the basic theorems of matrix arithmetic and differential geometry and I also applied the solution methods of the system of ordinary first-order inhomogeneous linear differential equations.

The simulated network configuration was implemented by a well-known and widely used simulation tool.

4 Summary of the Theses

1. I showed that congestion waves are formed naturally in the data traffic of computer networks in the direction opposite to the actual traffic. I pointed out that the intrinsic properties of the TCP protocol contribute to the formation and the stability of the transition of congestion. I pinpointed the large rate variation of TCP sending rate (burst effect) as one of the major contributors of this phenomenon. The mechanism behind the wave formation is that due to bursty sending of TCP, packet losses occur most likely in computers nearest to the site of the actual congestion. Other computers sharing the congested link increase their sending rates, moving the site of the congestion one site downstream. I checked this mechanism in a computer network with simple topology realized in a network simulator and I pointed out clear dependence between congestion transition and burstiness.

2. I proposed two models to investigate the speed, latency and network impact of short TCP downloads (typically appearing in Web applications). The first one is a simple model of multiple TCP connections sharing a link. Based on the transient behaviour of TCP, I derived an expression for the link utilisation as the function of the number of TCP connections. This expression serves as a basis for a Markov model describing the dynamics of the number of parallel TCP connections. Based on a simplified version of this model the main performance indicators of the traffic can be calculated. In the second model I investigated Web downloads where the objects of a Web-page are retrieved sequentially. I calculated the performance indicators (average transfer rate, average download time) in two cases: 1) the short files are downloaded sequentially, each of them opening a new TCP connection; 2) the TCP connection remains open for the whole download of the page (”pipelining” algorithm). I derived an expression to connect the calculated transfer rates in the above two cases. The calculations are validated both by simulations and real traffic measurements. Based on simulations I showed that the model is insensitive to the distribution of file sizes.

3. I investigated a queuing system with arrivals according to Markov Arrival Process (MAP), general service time and infinite servers ($MAP/G/\infty$ queuing system). The purpose was to determine the time-dependent moments of the queue-length. According to the literature there exists numerical solution to this problem. I showed that the differential equations for the time-dependent moments can be solved exactly if we restrict the problem to the case of Phase-type (PH) service time distribution ($MAP/PH/\infty$ queuing system). I demonstrated how this solution can be applied directly to the design of networking systems with parallel servers.

4. I investigated the probability density functions written as the sum of three exponentials where the coefficients in the exponents are real (i.e. their Laplace-transform has three distinct real poles). This is a special case of matrix analytic functions. I worked out a mapping between this type of functions and the two-dimensional vector space and the appropriate mapping of transformations. Using the invariant polytope approach known from

literature and differential geometry I showed a recursive method to find all functions on the two-dimensional plane that can be represented by matrices of order 1,2 or 3. These sets of functions correspond to invariant polytopes on the two-dimensional plane. The method can be applied to the functions that can be represented by more than 3 dimensions but these are not polytopes anymore but convex sets on the plane bordered by parametric curves. Moreover, I stated and proved a theorem for the generalised problem with n distinct real poles. In the end I showed a simple method to find a representation if an invariant polytope is given.

5 Conclusions

- In certain conditions the congestion control algorithms in TCP and the large fluctuations in the packet rate have the effect that induces the transition of congestion in computer networks, where the direction of the congestion waves is opposite to the actual traffic.

- The time-dependent moments of the queue length of a queuing system where the arrivals and departures are described by matrix analytic functions and the number of servers is infinite, can be obtained exactly. Using the construction and representation of matrix analytic functions the solution can be directly applied to modeling and design of telecommunication systems.

- Mapping a given group of matrix analytic functions (having 3 distinct real poles in their Laplace-transform) to the two-dimensional vector-space these distributions can be classified according to complexity and the minimal-order upper-triangular matrix representation of a given distribution can be obtained.

Publications related to the theses

Journal papers and book chapters

[J1] J. Stéger, P. Vaderna and G. Vattay, ”On the Propagation of Congestion Waves in the Internet,” *Physica A: Statistical Mechanics and its Applications*, Vol. 359, 784-792, January 2006.

[J2] P. Vaderna and T. Éltető, ”Transient Analysis of a Queuing System with Matrix-Geometric Methods,” *Lecture Notes in Computer Science, Next Generation Teletraffic and Wired/Wireless Advanced Networking* Vol. 4003/2006, 21-33, 2006.

[J3] T. Éltető and P. Vaderna, ”Időben változó sorbanállási rendszer vizsgálata mátrix-geometrikus eljárások segítségével,” *Híradástechnika* 2005/11, 38-44, 2005.

[J4] T. Éltető and P. Vaderna, ”Finding upper-triangular representations for Phase-type distributions with 3 distinct real poles,” *Annals of Operations Research* 160/1, 139-172, April, 2008.

Conference papers

[C1] T. Éltető, P. Vaderna and S. Molnár, ”Performance Analysis of TCP Networks Loaded by Web Traffic,” *18th International Teletraffic Congress, (ITC18)*, (Berlin, Germany), 2003.

[C2] P. Vaderna, E. Strömberg and T. Éltető, ”Modelling Performance of HTTP/1.1,” *IEEE Global Telecommunications Conference (GLOBECOM 2003)*, (San Francisco, CA), 2003.

Other publications

[O1] J. Stéger, P. Vaderna and G. Vattay, ”On the Propagation of Congestion Waves in the Internet ,” *The European Conference of Complex Systems, (ECCS’05)*, pid-128, (Paris, France), 2005.

[O2] P. Vaderna and T. Éltető, ”Transient Analysis of a Queuing System with Matrix-Geometric Methods,” *NEW2AN 2006*, LNCS 4003, 21-33, (Saint-Petersburg), 2006.

[O3] T. Éltető and P. Vaderna, ”Finding upper-triangular representations for Phase-type distributions with 3 distinct real poles,” *MAM6* (Beijing), 2008.

[O4] D. Krupp, G. Pongrácz and P. Vaderna, ”Regulation method of load on telecommunication network node e.g. media gateway, involves regulating load of slave node using preset leaky bucket restrictor and determining roundtrip response delay of messages sent from master node,” Patent Application WO-2008/ 043398-A1, 2008.