Theses of the PhD Thesis

Interior point algorithms for general linear complementarity problems

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Introduction

The thesis deals with the interior point methods for linear complementarity problems. Consider the *linear complementarity problem* in the standard form: find vectors $\mathbf{x}$, $\mathbf{s} \in \mathbb{R}^n$, which satisfy the constraints

$$-M\mathbf{x} + \mathbf{s} = \mathbf{q}, \quad \mathbf{x}\,\mathbf{s} = \mathbf{0}, \quad \mathbf{x},\, \mathbf{s} \geq \mathbf{0}, \tag{1}$$

where $M \in \mathbb{R}^{n\times n}$ and $\mathbf{q} \in \mathbb{R}^n$. The LCP is an $\mathbb{NP}$-complete problem [1], when we have no information about the coefficient matrix of the problem. In the literature there are efficient algorithms to solve the LCP when the matrix of the problem belongs to a certain special matrix class, for instance, to the class of copositive matrices for Lemke algorithm, or to the class of sufficient matrices for the criss-cross algorithm, or to the class of $\mathcal{P}_*$-matrices for interior point algorithms (the last two classes are proved to be the same). The decision problems related to copositive and to sufficient matrices – whether the matrix belongs to these matrix classes – are also $\mathbb{NP}$-complete problems [5, 9]. Consequently of this, an arbitrary LCP problem can not be solved in polynomial time, not even if the coefficient matrix belongs to one of the aforementioned class, since it can not be decided in polynomial time.

Our aim was to construct an algorithm, which provides some kind of information about the LCP problem – it gives a solution in the best case. We take well-known interior point methods (for LCPs with $\mathcal{P}_*(\kappa)$-matrices) for the basis of the new algorithm. The modified algorithms are polynomial time methods.

Our motivation was not only to handle the LCP problems in applications efficiently, but it was also theoretical. Based on the modified interior point methods we gave constructive proofs for new EP theorems.

The thesis can be divided into two parts. In the first part we mainly collected well-known results according to the LCPs and interior point methods. Our aim was to give an overview on the theory, which is the basis of the own results described in the second part of the thesis. At the end of the Introduction we collected some well known problems, which can be reformulated as an LCP to illustrate how important an efficient algorithm for handling LCPs with arbitrary matrices will be in practice.

In the second chapter we dealt with some matrix classes related to LCPs, which are important for our purposes. Here some observations according to $\mathcal{P}_*(\kappa)$-matrices are own results. We showed that the eigenvalues of the matrix do not determine this property, because there is a matrix, which belongs to this class, but we gave an other matrix with the same eigenvalues, which is not sufficient, even not $\mathcal{P}_0$. Furthermore, we examine the function

$$\kappa(\mathbf{x}) = -\frac{1}{4}\frac{\mathbf{x}^T M\mathbf{x}}{\sum_{i\in\mathcal{I}_+} x_i(Mx)_i}, \qquad \Big(\mathcal{I}_+ = \{i : x_i(Mx)_i > 0\}\Big)$$

which assigns the lower bound coming from the definition to all points $\mathbf{x}$, i.e., it gives a lower bound on the handicap of the matrix. The interior point methods use the $\mathcal{P}_*(\kappa)$ property only locally, therefore they do not discover the lack of this property, if in each iteration we use a direction for which this bound is finite. We showed that the function $\kappa(\mathbf{x})$ is not continuous, not even if the matrix is sufficient. Furthermore, if there is a point where this function is not defined, then the matrix is not column sufficient, and if the matrix is column sufficient, but not row sufficient, then there is a sequence of points where the function tends to infinity. The third chapter is an overview of the theory of interior point methods.

The dual linear complementarity problem

Let us now consider the *dual linear complementarity problem* (DLCP) [2]: find vectors $\mathbf{u}$, $\mathbf{z} \in \mathbb{R}^n$ which satisfy the constraints

$$\mathbf{u} + M^T\mathbf{z} = \mathbf{0}, \quad \mathbf{q}^T\mathbf{z} = -1, \quad \mathbf{u}\,\mathbf{z} = \mathbf{0}, \quad \mathbf{u},\, \mathbf{z} \geq \mathbf{0}. \tag{2}$$

The set of feasible solutions of the DLCP is

$$\mathcal{F}_D = \{(\mathbf{u},\mathbf{z}) \geq \mathbf{0} \;:\; \mathbf{u} + M^T\mathbf{z} = \mathbf{0},\; \mathbf{q}^T\mathbf{z} = -1\}.$$

We proved that if the coefficient matrix of the problem is row sufficient, then the set of feasible solutions and the set of complementary feasible solutions is the same for the DLCP problem.

Lemma 1 *Let the matrix M be row sufficient. If $(\mathbf{u},\mathbf{z}) \in \mathcal{F}_D$, then $(\mathbf{u},\mathbf{z})$ is a solution to the DLCP.*

The set of feasible solutions is determined by linear constraints. It is known that linear programming problems can be solved in polynomial time, for example with interior point methods, therefore we got the following complexity result.

Corollary 2 *Let the matrix M be row sufficient. Then the DLCP can be solved in polynomial time.*

This result enables us to formulate the following EP theorem and to give a constructive proof of it.

Theorem 3 *Let the matrix $M \in \mathbb{Q}^{n \times n}$ and the vector $\mathbf{q} \in \mathbb{Q}^n$ be given. Then it can be shown in polynomial time that at least one of the following statements holds:*

(i) *the DLCP has a feasible complementary solution $(\mathbf{u}, \mathbf{z})$, whose encoding size is polynomially bounded.*

(ii) *the LCP has a feasible solution, whose encoding size is polynomially bounded.*

(iii) *the matrix M is not row sufficient and there is a certificate whose encoding size is polynomially bounded.*

In the literature there are some EP theorems related to the LCP. The first was given by Fukuda, Namiki and Tamura [3], in addition Csizmadia and Illés provided a constructive proof for the LCP duality theorem in EP form [2]. However, these results are based on the criss-cross algorithm, therefore only the finiteness, i.e., the existence can be proved, while the polynomial time given answer can be guaranteed in our case. Let us remark that the three statements of our EP theorem is not exactly the same as the three cases of the mentioned other two theorems. In the second case of the other EP theorems a complementary feasible solution is given for the LCP, too, while in the third case instead of row sufficiency there is a stronger condition: sufficiency. We examine the problem from the dual side. In the first case we solve the DLCP, in the second case the feasible solution of the LCP is an evidence that the DLCP has no solution, but we can not say anything about the solvability of the LCP. In the third case we get information about none of the problems.

Linear complementarity problems for sufficient matrices

We generalized a version of Mizuno–Todd–Ye predictor-corrector algorithm for LCPs with $\mathcal{P}_*(\kappa)$-matrices in Chapter 5. In this chapter we assume that a feasible interior point is known and in addition, a nonnegative κ is known, with which the matrix of the LCP is a $\mathcal{P}_*(\kappa)$-matrix. This chapter is based on the paper of Potra [7], he established and analyzed the Mizuno–Todd-Ye algorithms for LCPs with positive semidefinite matrices. The Mizuno–Todd–Ye predictor-corrector algorithm takes a predictor, namely an affine step – where the aim is to decrease the duality gap as much as possible – and a corrector, namely a centering step – where we return to the appropriate neighbourhood of the central path with a full Newton step – alternately. This can be guaranteed by the suitable tuning of the two central path neighbourhoods. The key issue of the generalization of the algorithm was the determination of the proximity parameters, which guarantee the above property of the algorithm. The following central path neighbourhood was considered:

$$\mathcal{N}(\tau) := \left\{ (\mathbf{x}, \mathbf{s}) \in \mathcal{F}^+ : \left\| \sqrt{\frac{\mathbf{x}\mathbf{s}}{\mu}} - \sqrt{\frac{\mu}{\mathbf{x}\mathbf{s}}} \right\| \leq \tau \right\},$$

where $\mathcal{F}^+$ is the set of interior points of the LCP. Denote τ the proximity parameter of the smaller, while $\hat{\tau}$ the proximity parameter of the larger neighbourhood, i.e., $\tau > \hat{\tau} > 0$. We proved that if the proximity parameters satisfy the inequality

$$\hat{\tau} < \frac{2\sqrt{\tau}}{\sqrt{1+4\kappa}\sqrt[4]{2+\tau^2}}, \tag{3}$$

then the above property of the algorithm holds. Based on this, the following upper bound can be derived for the proximity parameter of the larger neighbourhood.

Lemma 4 *If* $\tau < \sqrt{-1 + \sqrt{1 + \frac{16}{(1+4\kappa)^2}}}$, *then there exists a positive* $\hat{\tau}$ *that satisfies (3) and the full Newton step is feasible at the corrector step.*

This lemma throws light on the weakness of the generalized Mizuno–Todd–Ye predictor-corrector algorithm. The value of κ effects the neighbourhood parameters τ and $\hat{\tau}$. The larger the value of κ is, the smaller the τ and $\hat{\tau}$ neighbourhoods are, which ensures that the Mizuno–Todd–Ye predictor-corrector algorithm takes one predictor and one corrector step alternatively. Therefore, if the κ increases, then the complexity of the algorithm declines.

One of the main steps of the interior point methods complexity analysis is computing a lower bound on the Newton step length. In our case, as we have already mentioned, a full Newton step is taken in each corrector step, so we only need to examine the predictor step length.

Theorem 5 *In the predictor step the maximal feasible step length* θ^* *satisfies* $\theta^* \geq \frac{\chi_n}{\sqrt{n}}$, *where*

$$\chi_n = 2\sqrt{\frac{\gamma}{m\left(\frac{\tau}{\sqrt{n}}\right)(1+4\kappa)}}\left(\sqrt{\frac{\gamma}{n\,m\left(\frac{\tau}{\sqrt{n}}\right)(1+4\kappa)}+1} - \sqrt{\frac{\gamma}{n\,m\left(\frac{\tau}{\sqrt{n}}\right)(1+4\kappa)}}\right) \tag{4}$$

is a bounded quantity.

Considering the above lower bound on the step length, an estimation can be proved on the duality gap, and so we can gave an iteration bound of the algorithm. We presented two suitable proximity parameters and the related complexity of the algorithm.

Theorem 6 *Let the LCP for any* $\mathcal{P}_*(\kappa)$*-matrix* M *be given, where* $\kappa \geq 0$ *and let* $\mu_0 = 1$, $\tau = \frac{1}{1+4\kappa}$ *and* $\hat{\tau} = \frac{\sqrt{2}}{1+4\kappa}$. *Then the Mizuno–Todd–Ye algorithm generates an* $(\mathbf{x}, \mathbf{s}, \mu)$ *point satisfying* $\mathbf{x}^T\mathbf{s} < \varepsilon$ *in at most* $O((1+\kappa)^{\frac{3}{2}}\sqrt{n}\log\frac{n}{\varepsilon})$ *iterations.*

Linear complementarity problems for arbitrary matrices

Our main results are presented in the last chapter. In the previous chapter we made two assumptions: 1. an initial interior point is known; 2. the coefficient matrix is a $\mathcal{P}_*(\kappa)$-matrix with an a priori known $\kappa \geq 0$. The first assumption is mainly technical, because we can either use an infeasible IPM, or the problem of an initial interior point can be handled by an embedding technique. However, the second assumption, the a priori knowledge of the parameter κ is too strong. There is no known polynomial time algorithm to check, whether a matrix is a $\mathcal{P}_*(\kappa)$-matrix or not. Potra and Liu relaxed this assumption [8], they modified their IPM in such a way that we only need to know the sufficiency of the matrix. However, this is still a condition that can not be verified in polynomial time, as we have already mentioned it in the introduction.

Our aim was to modify interior point algorithms in such a way, that they run for LCPs with any arbitrary matrix in polynomial time. Since the LCP is an $\mathbb{NP}$-complete problem, we can not expect a polynomial time algorithm, which can solve each LCP problem. Therefore, our algorithms terminate with one of the following cases: they either solve the LCP problem, or solve the dual problem (it proves that the LCP has no solution) or stop with a certificate, that the matrix of the problem is not a $\mathcal{P}_*(\tilde{\kappa})$-matrix with an arbitrary big, but with an a priori fixed $\tilde{\kappa} \geq 0$. Let us remark, that we speak about solutions, but these are only ε-optimal solutions (because of using interior point methods), where ε is an arbitrary small positive number and it is an upper bound on the duality gap.

Potra et al. [8] initially assume in their algorithm for LCPs with sufficient matrices that the matrix is $\mathcal{P}_*(1)$. At each iteration they check whether the new point is in the appropriate neighbourhood of the central path or not. In the latter case they double the value of κ. We use this idea in a modified way. The larger κ is, the worse the iteration complexity is, thus we only take the necessary enlargement of κ (until it reaches $\tilde{\kappa}$).

In IPMs the $\mathcal{P}_*(\kappa)$ property needs to be true only for the actual Newton direction $\Delta\mathbf{x}$ in various ways, for example, this property ensures that with a certain step size the new iterate is in an appropriate neighbourhood of the central path and/or the complementarity gap is sufficiently reduced. Consequently, if the desired results do not hold with the current κ value, we update the κ parameter to the lower bound determined by the Newton direction $\Delta\mathbf{x}$, i.e., $\kappa(\Delta\mathbf{x})$. If there exists such a vector $\Delta\mathbf{x}$ for which $\mathcal{I}_+(\Delta\mathbf{x}) = \emptyset$, and thus $\kappa(\Delta\mathbf{x})$ is not defined, then the matrix M of the LCP is not a $\mathcal{P}_*$-matrix. In this case we stop the algorithm, and the output will be $\Delta\mathbf{x}$ as a certificate to prove that M is not a $\mathcal{P}_*$-matrix.

There is another point where IPMs may fail if the matrix of the LCP is not $\mathcal{P}_*$. If the matrix is not $\mathcal{P}_0$, then the Newton system may not have a solution, or the solution may not be unique. If this is the case, then the actual point $(\mathbf{x}, \mathbf{s})$ is a certificate which proves that the matrix is not $\mathcal{P}_0$, so it is not $\mathcal{P}_*$ either.

To sum it up, we make three tests in our algorithms. In each test the property of the LCP matrix M is examined indirectly. When we inquire about the existence and uniqueness of the solution of the Newton system, we check whether the matrix is $\mathcal{P}_0$ or not. When we test some properties of the new point, for example whether it is in the appropriate neighbourhood of the central path or not, we examine the $\mathcal{P}_*(\kappa)$ property for the current value of κ. Finally, if the $\kappa(\Delta\mathbf{x})$ value is not defined, then the matrix is not $\mathcal{P}_*$. We note that at each step, all the properties are checked only locally, only for one vector of $\mathbb{R}^n$. Consequently, it is possible, that the matrix is neither a $\mathcal{P}_*$ nor a $\mathcal{P}_0$-matrix, but the algorithm does not discover it and solves the LCP in polynomial time, because those properties are true for the vectors $\mathbf{x}$ and $\Delta\mathbf{x}$ that were generated by the algorithm. It may also occur that the matrix is not $\mathcal{P}_*$, but the algorithm does not detect it. It only increases the value of κ if $\kappa < \kappa(\Delta\mathbf{x})$ and then it proceeds to the next iterate. This is the reason why we need the threshold $\tilde{\kappa}$ parameter that enables us to get a finite algorithm.

We modified three well-known interior point methods: the long-step path-following[6], the affine scaling[4] and the predictor–corrector interior point methods [8].

Denote $\bar{\theta}$ the current step length of the modified algorithm in each iteration. Furthermore, denote $\theta^*(\kappa)$ the special feasible step length, which was introduced in the complexity analysis of the corresponding original algorithm.

The long-step path-following interior point algorithm

In the modified version of the long-step path-following interior point algorithm we check in each iteration, whether the proximity measure, namely the distance from the central path, decreased sufficiently or not.

Lemma 7 *If after an inner iteration the decrease of the proximity is not sufficient, i.e.,* $\delta^2(\mathbf{xs}, \mu) - \delta^2(\mathbf{x}(\bar{\theta})\mathbf{s}(\bar{\theta}), \mu) < \frac{5}{3(1+4\kappa)}$, *then the matrix of the LCP is not* $\mathcal{P}_*(\kappa)$ *with the actual* κ *value, and the Newton direction* $\Delta\mathbf{x}$ *is a certificate for this fact.*

If the decrease is not appropriate, then we enlarge the value of κ on $\kappa(\Delta\mathbf{x})$. We verified that the modified long-step path following algorithm has the following complexity.

Theorem 8 *Let* $\tau = 2$, $\gamma = 1/2$ *and* $(\mathbf{x}^0, \mathbf{s}^0)$ *be a feasible interior point such that* $\delta_c(\mathbf{x}^0\mathbf{s}^0, \mu^0) \leq \tau$. *Then after at most* $\mathcal{O}\left((1+4\hat{\kappa})n \log \frac{(\mathbf{x}^0)^T\mathbf{s}^0}{\varepsilon}\right)$ *steps, where* $\hat{\kappa} \leq \tilde{\kappa}$ *is the largest value of parameter* κ *throughout the algorithm, the long-step path-following interior point algorithm either produces a point* $(\hat{\mathbf{x}}, \hat{\mathbf{s}})$ *such that* $\hat{\mathbf{x}}^T\hat{\mathbf{s}} \leq \varepsilon$ *and* $\delta_c(\hat{\mathbf{x}}\hat{\mathbf{s}}, \hat{\mu}) \leq \tau$ *or it gives a certificate that the matrix of the LCP is not* $\mathcal{P}_*(\tilde{\kappa})$.

The affine scaling interior point algorithm

In the modified affine scaling interior point method the reduction of the duality gap is inspected.

Lemma 9 *If* $\mathbf{x}(\bar\theta)^T\mathbf{s}(\bar\theta) > (1 - 0.25\,\nu\,\theta_a^*(\kappa))\,\mathbf{x}^T\mathbf{s}$*, that is, the decrease of the complementarity gap within the* $\delta_a \le \tau$ *neighbourhood is not sufficient, then the matrix* M *of the LCP is not* $\mathcal{P}_*(\kappa)$ *with the actual value of* κ*. The Newton direction* $\Delta\mathbf{x}$ *serves as a certificate.*

Similarly to the above, if the decrease is not adequate, then the value of κ is increased. In addition, we again proved the polynomiality of the modified algorithm.

Theorem 10 *Let* $(\mathbf{x}^0, \mathbf{s}^0) \in \mathcal{F}^+$ *such that* $\delta_a(\mathbf{x}^0\mathbf{s}^0) \le \tau = \sqrt{2}$*. Then after at most*

$$\begin{cases} \mathcal{O}\left(\frac{n(1+4\hat\kappa)}{1-2^{-\rho}}\log\frac{(\mathbf{x}^0)^T\mathbf{s}^0}{\varepsilon}\right), & \text{if } 0 < \rho \le 1 \text{ and } n \ge 4 \\ \mathcal{O}\left(n(1+4\hat\kappa)\log\frac{(\mathbf{x}^0)^T\mathbf{s}^0}{\varepsilon}\right), & \text{if } \rho = 1 \text{ and } n \ge 4 \\ \mathcal{O}\left(2^{2\rho-2}n(1+4\hat\kappa)\log\frac{(\mathbf{x}^0)^T\mathbf{s}^0}{\varepsilon}\right), & \text{if } 1 < \rho \text{ and } n \text{ sufficiently large} \end{cases}$$

iterations the affine scaling algorithm either yields a vector $(\hat{\mathbf{x}}, \hat{\mathbf{s}})$ *such that* $\hat{\mathbf{x}}^T\hat{\mathbf{s}} \le \varepsilon$ *and* $\delta_a(\hat{\mathbf{x}}\hat{\mathbf{s}}) \le \tau$*, or it gives a polynomial size certificate that the matrix is not* $\mathcal{P}_*(\tilde\kappa)$*, where* $\hat\kappa \le \tilde\kappa$ *is the largest value of parameter* κ*.*

The predictor-corrector interior point algorithm

As we have already mentioned, the predictor-corrector algorithm has two types of steps. Accordingly, there are two tests related to the Newton step length in the modified predictor-corrector interior point method. At first, the predictor step length is checked.

Lemma 11 *If* $\bar\theta < \theta_p^*(\kappa)$*, then the matrix* M *is not a* $\mathcal{P}_*(\kappa)$*-matrix and the affine Newton direction is a certificate for this.*

In the corrector step we examine whether we can return with the step length $\theta_c^*(\kappa)$ to the appropriate neighbourhood of the central path or not.

Lemma 12 *If* $\theta_c^*(\kappa)$ *is such a corrector step length that* $(\bar{\mathbf{x}}(\theta_c^*(\kappa)), \bar{\mathbf{s}}(\theta_c^*(\kappa))) \notin \mathcal{D}(\gamma)$*, then the matrix* M *is not a* $\mathcal{P}_*(\kappa)$*-matrix and the corrector Newton direction is a certificate for this.*

Finally, we also presented the complexity of the third modified algorithm, which is polynomial as well.

Theorem 13 *Let* $(\mathbf{x}^0, \mathbf{s}^0) \in \mathcal{F}^+$ *such that* $(\mathbf{x}^0, \mathbf{s}^0) \in \mathcal{D}(\gamma)$. *Then after at most*

$$\mathcal{O}\left((1+\hat{\kappa})n \log \frac{(\mathbf{x}^0)^T \mathbf{s}^0}{\varepsilon}\right)$$

steps, where $\hat{\kappa} \leq \tilde{\kappa}$ *is the largest value of parameter* κ *throughout the algorithm, the predictor-corrector algorithm generates a point* $(\hat{\mathbf{x}}, \hat{\mathbf{s}})$, *such that* $\hat{\mathbf{x}}^T \hat{\mathbf{s}} \leq \varepsilon$ *and* $(\hat{\mathbf{x}}, \hat{\mathbf{s}}) \in \mathcal{D}(\gamma)$ *or provides a certificate that the matrix is not* $\mathcal{P}_*(\tilde{\kappa})$.

Based on the modified interior point methods and their complexity results above, we could present the following EP type theorem.

Theorem 14 *Let an arbitrary matrix* $M \in \mathbb{Q}^{n \times n}$, *a vector* $\mathbf{q} \in \mathbb{Q}^n$ *and a point* $(\mathbf{x}^0, \mathbf{s}^0) \in \mathcal{F}^+$ *with* $\delta_c(\mathbf{x}^0\mathbf{s}^0, \mu^0) \leq \tau$ *be given. Then one can verify in polynomial time that at least one of the following statements holds*

(1) the LCP has an ε*-optimal solution* $(\mathbf{x}, \mathbf{s})$ *whose encoding size is polynomially bounded.*

(2) the matrix M *is not in the class of* $\mathcal{P}_*(\tilde{\kappa})$ *and there is a certificate whose encoding size is polynomially bounded.*

We can state our main result eliminating the interior point assumption from the theorem above. It also has a constructive proof. First, the dual problem is tried to be solved. When the second case of Theorem 3 is realized, we apply one of the above modified algorithms on an embedded model.

Theorem 15 *Let an arbitrary matrix* $M \in \mathbb{Q}^{n \times n}$ *and a vector* $\mathbf{q} \in \mathbb{Q}^n$ *be given. Then one can verify in polynomial time that at least one of the following statements holds*

(1) the LCP problem has an ε*-optimal solution* $(\mathbf{x}, \mathbf{s})$ *whose encoding size is polynomially bounded.*

(2) the DLCP problem has a feasible complementary solution $(\mathbf{u}, \mathbf{z})$ *whose encoding size is polynomially bounded.*

(3) the matrix M *is not in the class* $\mathcal{P}_*(\tilde{\kappa})$.

Theorem 15 is a generalization of Theorem 14. Since the interior point assumption is eliminated, it can occur that the LCP has no solution while the matrix M is sufficient. This is the second statement of Theorem 15. On account of the embedding model we only have an indirect certificate in a case of (3). This is the reason why in the last statement of Theorem 15 we can not ensure an explicit certificate. Therefore, Theorem 15 is stronger than Theorem 14, because the interior point assumption is eliminated, however only an indirect certificate is provided in the last case. We again emphasize, that the strength of this EP theorem lies in the polynomial time given answer.

At the end of the thesis we presented preliminary computational results of the modified long-step path-following and the predictor-corrector interior point methods on a special variant of Arrow–Debreu market equilibrium problem. We compare the efficiency of the modified methods and the algorithms presented by Ye et al. [10]. The modified algorithms are competitive for small scale problems (dimension less than 100). The most important advantage of modified interior point algorithms is that they can determine different solutions. Furthermore, we do not use the special structure of the problem, only at generating initial points, but it can be substituted by an embedding.

There are some possibilities to improve the modified interior point algorithms, for example, a sophisticated initial point generation, or using the information of previous runs. On the theoretical part, we would like to examine the Rounding procedure of the interior point algorithms (how to get an exact solution from an ε-optimal solution) for arbitrary matrices, give an EP theorem according to this as well, so the "ε-optimal solution" could be changed to "optimal solution" in Theorem 14 and 15.

The thesis is based on the following papers

[1] T. Illés and M. Nagy. A Mizuno–Todd–Ye type predictor-corrector algorithm for sufficient linear complementarity problems. *European Journal of Operation Research,* 181(3):1097–1111, 2007.

[2] T. Illés, M. Nagy and T. Terlaky. An EP theorem for dual linear complementarity problems. *Journal of Optimization Theoryan Applications,* 140(2):233–238, 2009. http://www.springerlink.com/content/w3x6401vx82631t2/fulltext.pdf

[3] T. Illés, M. Nagy and T. Terlaky. A polynomial path-following interior point algorithm for general linear complementarity problems. *Journal of Global Optimization,* In print, accepted: 26 August 2008.
Online version: http://www.springerlink.com/content/547557t2077t2525/

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