THEORETICAL MODELING OF DC ELECTRIC WELL LOGGING TOOLS IN INHOMOGENEOUS MEDIA
PhD thesis’s

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**Introduction**

The resistivity data originated from electric measurement have a basic role in the well log interpretation. Because of the large investigation depth, the resistivity tools are sensitive of the different inhomogenities (borehole, invaded zone, random inhomogenities etc.) which may strongly affect the measuring data.

The main subject of this thesis’s the more accurate theoretical modeling of the DC well logging tools in order to enhance the estimation of resistivity parameters (inversion). The main area of enhancement:

- *Electrode model.* In the solution of direct problem, a realistic electrode model was applied (de-Witte model), which is suitable for the surface current distribution calculation.

- *Study of radial inhomogeneity (invasion profile).* In the conventional direct problem solution the step-function is applied as a radial resistivity profile, which sometimes is a poor approximation of real invasion profile. In this dissertation the new calculation method is presented for the case of more realistic radial resistivity profile agreeing with the results of laboratory measurement.

- *Study of vertical inhomogeneity.* The calculation method was elaborated to solve the direct problem in the case of vertically varying resistivity.

- *Study of random inhomogenities.* The effect of random inhomogenities is a special model error which is not taken into account during the interpretation, so the study of it necessary for the analysis of inversion.

**Antecedents**

The problems outlined above have been in the focus of the research related to the direct and inverse problem for a long time.

The electrode model is one of the most important element of direct problem. In my calculation the ring electrode model, suggested by de-Witte [1959], was the starting point for the modeling of different electrode systems. In this approximation the potential field of ring electrode can be regarded as a Green-function to calculate the potential field of compound electrode system by convolution.

In conventional calculation the inhomogenities are approximated by step function hence the well known solutions of Laplace equation can be applied. The accepted way of invasion correction is also based on the step function profile which is rough approximation of real resistivity profile measured by Jiao [1992]. The direct problem solution in the case of stepwise inhomogenities is known from the papers of Dakhnov [1967], Gianszero [1982], Anderson [2001]. The recursive solution was determined for vertical inhomogenities by Daniels [1971] and for radial inhomogenities by Drahos [1984] for multilayered media. In the paper of Roy [1994] approximating solution was derived for the case of linear radial invasion profile by Frobenius-method.

In the literature lots of methods are presented for the equivalent resistivity calculation (homogenization) in the case of random media. First of all, the methods based on the perturbation theory [Landau 1986;Teodorovich 2002] or variation principle [Hashin 1962;Alliere 2003] have to be mentioned.
Applied methods

In the case of DC tools, the direct problem solution is essentially the solution of the Laplace equation under the proper boundary condition. At first the Green-function has to be determined, and then an integral equation can be derived for the current density distribution of electrode system. The current densities were calculated by the discretization of integral equation and finally the potential field was calculated by convolution of Green function and current distribution. It should be noted that the current distribution of measuring electrodes were also calculated [Balázs 1987, 2005].

In the case of continuous resistivity distribution, the differential equation system was derived for the coefficient function (spectra) of general solution. The general solutions were given in the Bessel-Fourier (for radial variation) or Hankel (for vertical variation) transformed form. The potential function was determined as a solution of differential equation system by numerical method and approximating analytical form. [Balázs 2007] Similar solution was achieved by handling the radial variation by perturbation method. The results of modified direct problem are used in inverse problem.

The effect of random inhomogenities is also studied by perturbation method in different order. The main goal was the equivalent resistivity determination. The results in different order can be calculated by recursion using the basic solution of Laplace equation.

Results (PhD thesis’s):

1. Based on the de Witte electrode model, a method was elaborated, which is suitable for the calculation of the finite cylindrical electrode potential field and surface current distribution in the various media with radial inhomogenities. The method applicable both in the case of current and both of the measuring electrodes, therefore potential field of any kind of electrode system can be calculated. Using the results, the direct problem of penetration electric tool and laterolog has been solved. In the case of penetration tool first of all the cone and shaft effect was examined, in the case of laterolgs the surface current distribution was studied in the prescribed current regulation condition.

2. The possibilities of DC tools direct problem solution were studied in different approximation for radially inhomogeneous media. Using the results of study, a calculation method was elaborated. In this method the differential equation system was determined for spectral coefficient functions which appear in the general solution given in Bessel-Fourier transformed form. The measured media was divided to radially infinitesimal cylindrical zones and the system of equation was derived from boundary conditions. In this approximation each cylindrical zone takes effect for the coefficient function vector as an infinitesimal transformation. The method was applied in the case of linear resistivity profile, and the results were compared with the results of conventional profile calculation, presenting the differences and equivalences.
3. Using the results originated from the direct problem solution of radially inhomogeneous case, resistivity inversion was performed for a normal log combination measured in shale-sandstone series. The results are compared with the results of conventional inversion. From the comparison it can be concluded that the increment of invasion increase the deviation between the resistivity parameters estimated by different resistivity profile assumption. In general the square error of parameter fitting was less in the case of linear profile than the case of step profile, which is confirm the justification of enhanced invasion model.

4. In the case of vertically inhomogeneous media, a differential equation system was derived for the spectral coefficient functions perform in the general solution expressed in Hankel-transformed form. The differential equation was solved in the case of continuous vertical resistivity variation. The solutions were tested in simple cases where the exact solution is available. With this method the behavior of electrical logs can be studied in vertical inhomogenities.

5. In the apparent resistivities measured by electric tools the effect of random inhomogenities also appears. The recursive perturbation calculation method was elaborated to calculate the potential field formed in random media. The main goal of this simulation was the study of equivalent resistivity and its variance.

Conclusions

Finite electrode modeling

Studying the finite current electrodes, it can be concluded, that a specific current density distribution profile formed on the source electrode surface, which is increase towards the electrode edge (approximating mathematical formula is given in the dissertation). The current density profile deviation from the uniform distribution increases with the near resistivity contrast. In the case of measuring electrodes asymmetric current distribution is formed, that is on the higher potential part current enters to the electrode and on the lower potential part current is emitted.

In the compound electrode system the surface current distributions of individual electrodes are coupled depending on the near resistivity contrast. The current density distribution reacts to the regularization. For the dual-laterologs, correction diagram was calculated which slightly differs from the diagram published by the producer. In the case of penetration electric tools the effect of shaft and cone was calculated and it was shown that this effect reduce the potential at the measuring electrode about 10 %.

Continuous radial resistivity profile

In the examples of my paper, the resistivity profile was approximated by linear profile in the invaded zone. To demonstrate the calculation method, the measurements of Gulf-coast normal sonde combination was selected, which was often applied earlier in Hungarian hydrocarbon research and now has a great importance in the reevaluation of old gas field. It can be concluded, that the conventional and new inversion method
give a similar results in the case of smaller invasion, but at deeper invasion region the
conventional method overestimate the Rt parameter and the hydrocarbon saturation.

Vertically varying resistivity

Layers with continuously varying resistivity often can be found in sedimentary series. The
calculation method described in chapter 6 is suitable the study the electrical log
response in these environments.

Effect of small resistivity perturbation – random media

The effect of resistivity inhomogenities was studied in the cases where the
inhomogenities were located randomly or as an equidistant grid point in the measured
media. The potential distribution was calculated by perturbation method (till third
order). The fluctuation of potential (variance) was also examined as a function of
distance between the source and the measuring point. In the example shown in my
paper, the relative fluctuation decreases sharply with the distance of electrodes. If the
distance was 15 times longer than the measure of fluctuations, the relative variation
was under 1 %.

Connecting publication:

Balázs, L., 2007; Analytic approximation of Green-function in well logging electric direct
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