Finite size effects in two-dimensional quantum field theories

In the folded, double frequency and supersymmetric variants of the sine–Gordon model

Theses

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Introduction

Theoretical investigation of quantum field theories is usually carried out in infinite volume, as the lack of boundary conditions or the availability of a simple base means substantial help deriving the solutions. In the laboratory, however, we can not afford using arbitrarily large samples, or we may want to investigate a particular phenomenon in small scales or determine its volume dependency. So from the practical point of view it is fairly important to gain a clear picture of the finite size effects of quantum field theories [1, 2].

Finite size effects play an important role in lattice field theory for a similar reason. These days it is technically impossible to achieve lattice sizes large enough for direct measurement of S-matrices via asymptotic states for example. This makes lattice field theory a promising application of methods able to give scattering amplitudes from the volume dependency of some quantity.

However, exact solutions of generic $3 + 1$-dimensional interacting quantum field theories are usually not available to us even in infinite volume, apart from perturbative expansions of certain quantities. This complicates the investigation of finite size effects, since it excludes the especially interesting nonperturbative phenomena from our view, and also makes the interpretation of the finite volume results harder, through the lack of a known baseline. For this reason the so called (fully) integrable models are particularly useful subjects of such investigations, because we possess exact analytical results on their spectra and scattering amplitudes.

This thesis is an investigation of three variants of the well-known sine–Gordon model, an integrable quantum field theory. The different variants require different approaches, depending on the integrability of the resulting model and whether it can be considered a perturbation of some other integrable model. Also, the set of applicable methods depends on whether the model is considered in finite or infinite volume.

The combined usage of truncated conformal space approach, form factor perturbation theory and perturbed conformal field theory results in a consistent picture. This extends our experience about the applicability of the different methods and proves some earlier conjectures as well. On the other hand, by connecting the various regimes of validity of the different methods we are also able to extract new information about these well-known models.

Methods

Truncated conformal space approach (TCSA) is a variational method for investigating the spectra of perturbed conformal field theories in finite volume. It does not depend on the integrability of the model, which makes it equally applicable for the above three deformations of the sine–Gordon model, but being a numerical approximation the interpretation of the results requires special care.

One can determine the finite volume spectra of integrable quantum field theories based on their infinite volume description as well. In large volume the massive particles are basically always far away from each other, so they can be regarded point-like and the volume dependency of the energy levels can be investigated by quantum mechanical methods using the known exact S-matrix.

For nonintegrable models similar results can be obtained via form factor perturbation theory (FFPT), if the particular model is a perturbation of an integrable one. In our case the double frequency sine–Gordon model satisfies this requirement, and it possesses two perturbative regimes even by exchanging the roles of the different frequency potential terms. However, the most interesting feature of the model, namely the presence of a phase boundary is out of reach for perturbative methods, so one has to resort to TCSA for its investigation.

Results

The investigation of the various deformations of the sine–Gordon model brought the following results:

- In the case of the k-folded sine–Gordon model I showed [10] that the change of boundary conditions does not spoil the renormalizability of the theory, but nevertheless changes the spectrum in a spectacular way. The k-dependent degeneracies and the volume dependence of the energy levels of multi particle states was correctly described by the formalism developed by Klassen and Melzer [3], indirectly proving the S-matrix conjecture. I also measured the expectation values of the exponential fields in the various vacua of the model, providing evidence for the exact formula proposed by Lukyanov and Zamolodchikov [4].

- For the double frequency sine–Gordon model I first checked [11] the consistency of the FFPT and TCSA results in the two perturbative regimes of the model.

- Then I focused my attention on the case where the ratio of the two frequencies is 1/2. In this model Delfino and Mussardo predict a second order phase transition [5]. Contrary to the literature [6] I found that both lowest order correction terms to the effective potential are in principle capable of rendering the transition first order, although in this particular case it does not happen: within the (small) errors of TCSA I showed the transition to be of second order for all values of the coupling constant, and also identified it to belong to the Ising universality class. I found the UV-IR correspondence of states to agree with the previous conjectures [6].

- Applying more refined numerical analysis I excluded [12] the semiclassical results of Mussardo et al. [7] by two orders of magnitude. I also hinted on the problems of the derivation.

- In the context of the supersymmetric sine–Gordon model my first concern was the kink and vacuum structure [13], which is somewhat controversial in the literature. Adapting the argumentation of Zamolodchikov for the perturbed tricritical Ising model [8] I showed that the vacuum structure is indeed compatible with the S-matrix conjecture [9].

- I described the model in the perturbed conformal field theory framework, and proved that here — contrary to the Lagrangian description — one need not consider the scalar potential term.

- In this model the TCSA is plagued by UV divergences, but the results correctly describe the qualitative properties of the spectrum and the remaining precision is also enough to extract the mass ratios of the lowest excitations. Thus I was able to connect the lowest few IR energy levels with the conformal field theory operators creating them in the UV limit.

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