Evolutionary Prisoner’s Dilemma Game on Graphs

Theses

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2008.
**Introduction**

One of the central questions in evolutionary game theory is to find necessary conditions and mechanisms that result in cooperation among selfish individuals. In my thesis, I have investigated this question in the framework of the evolutionary prisoner’s dilemma game.

With the spatial extension of the prisoner’s dilemma game, it is possible to take into account the structure of links inside a community when one creates a new model. In these models, the community is described by a graph: the nodes of the graph define the members of the community (players), while the edges show the links of interactions between them. The members play prisoner’s dilemma games in pairs, and the payoffs from the different games make the total income of the player for that round. From time to time, players can adopt the strategy of another player with a probability depending on their payoff difference.

Such systems are well-fitted to be investigated with the tools of statistical physics which were originally developed for the study of complex systems. This can explain the big interest of statistical physicists for the problems of game theory.

Of course it is impossible to merge all relevant features into a single model because in this case the analysis of the smallest part of the parameter space would consume an incredibly big amount of time. As a consequence of this, I added only one or two relevant features to the prisoner’s dilemma at the creation of the models.

**Cooperation for volunteering and spatially/temporary random partnerships**

First, I began to study the recently introduced voluntary participation and the different, random neighborhoods. I handled the refusal of the participation in the game as a third strategy (beside the cooperator and defector strategies). This strategy helps to avoid the absorbing defector state for higher $b$ (temptation to defect) parameter values in the spatial prisoner’s dilemma game. Moreover, I investigated the effects of temporary and spatial randomness in the net of connections, but I kept fixed the number of neighbors to avoid probable problems arising from the fluctuation of the number of neighbors. The spatial randomness can model the long range links between the local communities.
while temporal randomness can describe the effect of rarely appearing random partners. In the first part of the thesis, I searched for the effects of changing the randomness parameters on the maintenance and/or spreading of cooperation while scanning the system from the absolutely ordered state (square lattice) to complete randomness (mean-field limit).

**Prisoner’s Dilemma Game on Fixed, Scale-Free, Hierarchical Structure**

In the next part, the net of connections was defined by a scale-free graph. This kind of graph can be frequently found in nature; among other things, usually human relationship systems follow this distribution. That is why this structure could be an ideal underlying graph for studying prisoner’s dilemma-like conflicts. Another relevant characteristic of these networks is the clustering coefficient which is much higher than that of random graphs, i.e. the members of the community gather in small well connected groups much more than in the case of random graphs. In the thesis, I aimed to reveal the connection between the measure of cooperation and the changing of the size of the structure (the number of hierarchical levels) and the parameter $b$. A more basic question is whether this structure is suitable for the survival of cooperation.

**Hierarchical Lattices**

As in the studied model environment the scale-free graphs could not guarantee the conditions for the survival of cooperation, I began to investigate another type of hierarchical structure. The behavior of the two-strategy (defector and cooperator), spatial prisoner’s dilemma game is well known on the square lattice therefore I created a multilevel network of this simple base-structure and studied the effects of sub- and superordination. The tasks related to this model were the following: to analyze the forming of cooperation on the different hierarchical levels; to study the connection between the colonies on neighboring levels; to investigate the effects of changing the number of hierarchical levels; and search for an optimal number of levels (if it exists).
Phase Diagrams of the Prisoner’s Dilemma Game

After having done with the investigation of hierarchical lattices, I started the study of the noise built into the dynamical rule. It turned out that the noise and the topology of the neighboring network can have a very big impact on the measure of cooperation. In certain cases, increasing the volume of the noise can considerably advance cooperation. It needed systematical research on several simple networks (two-dimensional lattices, random regular graphs, small world networks) to clear up the relevant, basic topological features promoting cooperation. For this reason, I had to draw a map of all such parameter-combinations on the $b$-noise plane where cooperation could survive.
Methods

I have used basically two approaches to analyze the models in my thesis. As an “experimental” approach, I applied Monte Carlo simulations. I chose the system sizes so that the finite size effects could have been neglected. Usually, I started the simulations from a random initial state, and after a transient time I recorded the time dependence of the different, relevant quantities. With the knowledge of these data, it was easy to calculate the derived quantities (averages, fluctuations, etc.). In some cases, the problem could have been solved with special initial conditions; of course, in these cases, the relevant quantities were recorded from the beginning.

In the “theoretical” approach, I analyzed the behavior of the systems with different approximate methods. First, I studied the predictions of the mean-field theory, but as this method does not fit very well for the spatial systems I had to use dynamical cluster approximations on clusters of different sizes and of shapes fitting to the underlying structure. In this approximation, differential equations are stated for the time evolution of all possible configurational probabilities on the clusters and they are integrated numerically to get the equilibrium values. For suitable cluster sizes, the dynamical cluster approximation and the Monte Carlo simulations gave qualitatively the same result in almost all the cases.
**Theses**

1a. I have studied the prisoner's dilemma game on square lattice with quenched, spatial random partnership and showed that for small randomness parameter values, the self-organizing pattern remains the same as it was observed on the unmodified square lattice.

1b. For medium parameter values, the system showed a globally oscillating phase.

1c. For further increasing the measure of randomness, the system reaches a homogeneous, absorbing state. The quality of the absorbing state depends on the initial conditions and the exact value of the randomness parameter.

1d. The effects of the temporary random partnerships are basically similar to those of the quenched, spatial randomness, but the system is more sensitive to the temporary randomness in the connections, the transitions (1b.-1c.) take place at smaller parameter values.

2. I have studied the prisoner's dilemma game on a fixed, scale-free, hierarchical structure and normalized the payoff of the players with the number of their neighbors. I have showed that in this framework, cooperation can survive for a very long time in small cliques formed exclusively by cooperators but due to the noise built into the strategy adoption rule, defectors will dominate the whole community after that period.

3a. I have showed that on the square lattice the concentration of cooperators vanishes according to a second order phase transition belonging to the directed percolation universality class.

3b. According to my simulative results the concentration of cooperators goes to zero on every hierarchical level at the same b (temptation to defect) parameter value on the hierarchical lattices.
3c. The measure of cooperation differs strongly on the different levels. For four or less than four hierarchical levels, the rate of cooperative players is the highest on the top level and it decreases monotonously going downwards on the structure. For more than four levels, this arrangement can be observed only in the vicinity of the critical point.

3d. The measure of cooperation is always the smallest on the lowest hierarchical level.

3e. For the studied noise parameter value, the total income of the community is the highest for the structure with four hierarchical levels.

4a. I have determined the phase diagram of the prisoner's dilemma game on the $b$-noise plane for several spatial and non-spatial structures with Monte Carlo simulations and dynamical cluster approximations as well.

4b. I have showed that in the low noise limit, cooperation is highly promoted if overlapping triangles percolate through the structure.

4c. In the high noise limit, the presence of loops hinders the spreading of cooperation therefore in this case, cooperation can survive on the random regular graphs in the widest parameter range.

4d. I have found resonance-like behavior as a function of noise in cases without triangle percolation.
Publications


