Physics Doctorate School

School leader: Prof. Zalán Horváth

Particle Physics and Astronomy Program

Program leader: Prof. Ferenc Csikor

Gravitational Waves Emitted by Compact Binary Systems

Thesis book

János Majár

Supervisor: Dr. Mátyás Vasúth

MTA KFKI RMKI Theory Department

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1 Introduction

With the construction of the ground based gravitational wave observatories the investigation of gravitational wave signals has been receiving new impetus in the last two decades. The description of the dynamics of the different astrophysical sources and the detectable signal of the emitted gravitational waves has become one of the main lines of the research in general relativity and astrophysics.

The most promising sources of gravitational waves which can be detected by these observatories are binary systems of compact objects. The coalescence of these binaries — which produces detectable signals with high amplitude — can be divided into three main epoch: the early, "inspiral" phase before the coalescence, the "merger" era which describes the coalescence itself, and the "ringdown" part.

The ground based detectors are sensitive mainly for the wave signals emitted in the "merger" era, because these signals have appropriate amplitude and frequency properties to be detected. These kind of signals are described by numerical simulations.

For the detection of the gravitational waves the description of the first, "inspiral" part of the coalescence is also relevant. Because of the short duration of the merger it is important to determine the direction of the source as soon as possible. To do it one needs to describe the gravitational waveform in the "inspiral" epoch analytically. As a further advantage of this description is to give analytic initial conditions for the simulations modeling the coalescence.

But the main purpose of the determination of the detectable gravitational wave signal in the "inspiral" era is the LISA project, which aims to install a detector system into space. Because of the different size and technology LISA and the other, space based detectors will be most sensitive in the frequency range of the emitted wave signals by "inspiralling" compact binaries.

There are two, highly related methods used to describe the dynamics of the source and the emitted gravitational radiation in this era.

The post-Newtonian approximation is used to investigate the dynamics of the source. The essence of this approximation is the neglection of ultrarelativistic and extreme gravitational effects. Based on this approximation we use a series expansion, where the expansion parameter is ϵ the post-Newtonian parameter ($\epsilon \sim v^2$ and $\epsilon \sim M/r$). This approach can be used until the binary reaches the innermost stable circular orbit.

To describe the emitted gravitational waves we use the post-Minkowskian approximation, which is a formal series expansion in the Newtonian gravitational constant. Its main advantage is that a consistent multipolar expansion of the waveform can be given, where in the case of a post-Newtonian source the multipole moments can be expressed by the sum of terms corresponding to different post-Newtonian orders.

There is a region in spacetime where both methods are valid (the outer region of the source and the near zone of the radiation), and the theories can be matched together in this region. This way the theory of the metric perturbations created by the post-Newtonian source can be extended into the radiation zone, where we detect gravitational waves. Formally it means that the post-Newtonian form of the sources energy-momentum tensor arise in the multipole moments of the detectable waveform.

There are several aspects in the literature of the dynamics of the source investigated by the post-Newtonian approach. The description of the radial motion, namely the parameterization of the orbit is well known up to 3–3,5 PN order, although these results are valid under some simplifying conditions (neglecting the rotation of one or both of the objects, neglecting radiation reaction, or in the extreme mass ratio limit). The full description (including angle variables) of the motion is known only in special cases, namely up to 1,5 PN order in the test particle limit, and neglecting spin effects up to 3 PN order.

The result of the investigations of the gravitational wave research is primarily the form of transverse-traceless tensor describing metric perturbations formally in terms of the main dynamical quantities of the binary system (separation vector, relative velocity vector, spin vectors, masses) up to 1,5–2 PN order. The higher order contributions are valid in the non-spinning case up to 3–3,5 PN order.

There are some results giving the explicit time dependence of the detectable wave signal analytically, but these investigations are valid only in special cases (in the lowest, Newtonian order, in the case of circular orbits, one spinning body or equal masses). The main difficulty of the general description is that to project the transverse-traceless tensor to get the detectable wave signal one needs the full description of the dynamics of the source, including angle variables.

2 Main objectives and methods of the research

The main objective of the research is the analytic description of the dynamics and the detectable gravitational waveform of the emitted gravitational radiation of compact binary systems in the ”inspiral” phase with the consideration of the effects of the eccentricity of the orbit and the rotation of the bodies up to 1,5 PN order.

During the calculations I have used the post-Newtonian approximation for the investigation of the dynamics of the binary system and the multipole expansion of the post-Minkowskian approximation applied to post-Newtonian sources to describe the detectable gravitational waveforms. The effects of rotation and eccentricity can be studied in full detail and transparent results up to 1,5 PN relative order in this formalism. In this order the transverse traceless tensor representing metric perturbations and the main expressions of the parameterization of the orbit is given, which constitute the basis of my work.

I have discussed three physical systems during my research: first a test particle which orbits a rotating, massive body, second two comparable mass binaries with one spinning component, and the general case, when the orbiting bodies have comparable masses and spins. The Ph.D. thesis contains the detailed discussion of the test particle and general cases. The second, one spinning component case is obtained as a limit of the general description.

Since in the test particle limit in the Lense-Thirring approximation (which describes the dynamics as a geodesic motion in the gravitational field of a massive, rotating body) the precession of the spin vector is negligible and the full description of the motion is available I have chosen this system to test the method of the calculation of the detectable waveforms and to study the effects of the spin-orbit interaction.

In the general case I have considered the relativistic post-Newtonian corrections besides the spin-orbit interaction. Up to 1,5 PN all the limiting cases can be calculated from this general description.

I have investigated the physical system in two steps. Since the detectable gravitational waveform can be unambiguously expressed as a linear combination of polarization states h_+ and $h_\times$ in the first step the main objective was the construction of a method for the calculation of these polarization states which is independent of the parameterization of the orbit. The second step was the calculation of the explicit parameter dependence of the waveform with the help of the generalized true anomaly parameterization and the discussion of its structure. One of the most significant step is the description of the circular orbit limit, since in this case the results can be expressed explicitly in time.

3 Main results of the research

1. I have constructed a method to determine the form of the polarization states h_+ and $h_\times$ in the test-particle case including spin-orbit corrections in a way independent of the parameterization of the orbit.

   The basic step of the method is the introduction of the coordinate systems needed to integrate the equations of the motion and project the transverse-traceless tensor describing metric perturbations into the polarization states. To do it one needs the equations of the motion for the angular variables.[1]

   With the use of the constructed method given above I have evaluated the form of the polarization states of the detectable wave signal in terms of the true anomaly parameter. At first the description of the dynamics was completed with the integration of the equations of motion for the angle variables up to 1,5 PN order.

   In the second step, the parameter dependence of h_+ and $h_\times$ was given. The results contain higher harmonics according to the true anomaly parameter with constant coefficients depend on the constants of the motion, and the constant angles which arise from the geometry and the integration of the equations.

2. I have obtained the above result in the circular orbit case. Although the main steps of the method do not change, the assumptions of the circular orbits made the expressions much simpler.[1]

   With the integration of the equations of motion, and following the steps of the method, the form of the polarization states, the frequency of the basic harmonic and the form of the relevant higher harmonics can be given. These results in the Newtonian order are in agreement with previous works.

3. I have determined the system of the dynamical equations in the case of comparable masses and arbitrary spins. To do it I have given the dynamical equations for the angles describing the direction of the spin vectors. With the use of it and the radial equation of the motion the equations for the angular variables of the motion were given, and hence the description of the dynamics were completed up to 1,5 PN order.[2, 3]

   Similarly to the test particle case I have determined the method to evaluate the polarization states, which is independent of the parameterization of the orbit and includes the corrections of the spin-orbit interaction.

   The connection between the extreme mass ratio limit of the general description and the test particle case is investigated.

4. I have evaluated the parameter dependence of the polarization states of the detectable gravitational waves with the use of the true anomaly parameterization and the explicit time dependence in the circular orbit case.

   The spin precession equations and the equations of motion were integrated, and following the steps of the given method, the form of the polarization states were given. The results are similar to the test particle case, h_+ and $h_\times$ can be given as the sum of higher harmonics of the true anomaly parameter, however, the expressions for the coefficients are quite different.

   It can be seen that compared to the general description in the circular orbit case only some of the coefficients are non-zero, and hence at the Newtonian order our results reproduces the well-known fact that at this order the frequency of the detectable singal is twice the orbital frequency.

   The one spinning body and spinless cases arise when the length of the spin vectors go to zero. With the investigation of these limits one can find that the spin-orbit interaction gives contributions to the lower harmonics at each order.

References

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