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Fluctuations in strongly correlated equilibrium and nonequilibrium systems

Theses of the PhD dissertation

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Budapest, 2006.

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1. Introduction

The formation of steady states in systems far from equilibrium has been the focus of numerous studies, but a universal description comparable to equilibrium statistical physics, has not emerged so far. There have been though several approaches which had remarkable success and continue to receive particular attention. One of them is the investigation of nonequilibrium phase transitions, where the basic assumption is that the universality displayed by equilibrium phase transitions carries over to critical phenomena in the nonequlibrium case, as well. It is well known from the equilibrium theory of critical phenomena, that approaching a critical point the system behaves strongly correlated, implying that the distribution functions of macroscopic quantities usually become non-Gaussian, however they can be used to characterize the given universality class. There is a straightforward extension of these concepts to nonequilibrium systems. The advantage of studying the scaling functions associated with the distribution functions through finite size scaling is that the construction of the functions does not involve any fitting procedure, thus the comparison with the experiments can be performed without any uncertainty inherent in the fit parameters.

The aim of the first part of my work is the characterization of strongly correlated nonequilibrium systems by means of distribution functions. The first model system investigated is a quantum spin chain at zero temperature which is driven to produce an energy current. Besides the simplicity of this model, this choice is further motivated by the fact that the dynamics is generated naturally by the Schrödinger equation, thus we avoid the arbitrariness of defining the nonequilibrium dynamics. This model has a quantum phase transition into a current-carrying phase with long-range correlations, raising the question whether these states can be regarded as effectively critical ones. One expects to answer this question through the study of the distribution functions.

Another interesting field of research where the study of the scaling functions has been of great success is surface growth phenomena. In connection with this I have studied the surface fluctuations of an Edwards-Wilkinson type surface defined on a spherical substrate, which was motivated by a huge interest in understanding the temperature fluctuations of the cosmic microwave background radiation.

The second part of my thesis is concerned with a different approach to strongly correlated systems. At very low temperatures quantum mechanical effects will dominate the behaviour of the system, which might give rise to strong quantum correlations. A prominent example is the so-called entanglement phenomenon, which manifests itself as distinct parts of a system are described by a mixed state, even if the system as a whole is at a zero temperature ground state. Recently entanglement properties of equilibrium spin chains have been intensively investigated, and several interesting results have been obtained. In my thesis I investigate entanglement properties of a spin chain driven out of equilibrium by an energy current.

In the course of the investigation of entanglement properties I have addressed the question of a possible effective thermodynamical description of the mixed state of a subsystem. It is presented in the last part of my thesis through the example of a simple equilibrium XX chain.

2. Theses

The main results of the PhD dissertation can be summarized in the following theses:

1.) I have calculated analytically the distribution function of the transverse (non-ordering field) magnetization in both the equilibrium and nonequilibrium phases of the transverse Ising chain carrying an energy flux. I have found that the distributions are Gaussian even at the critical point of the system. The reason for this is that although the appropriate correlations decay with distance n as a power law but the exponent in the power is large ($1/n^2$), so that the fluctuations do not diverge.

2.) I have evaluated numerically the distribution function of the order parameter up to system sizes of 20 spins. The distributions are Gaussians in the equilibrium phases away from the critical point. At the critical point one has nontrivial scaling functions which depend strongly on the boundary conditions applied. The unexpected simplicity is in the current-carrying phase where the energy flux generates long-range correlations decaying as a power law ($1/\sqrt{n}$) but, nevertheless, the distributions are Gaussian, and the width of the Gaussians is decreasing with increasing energy flux. The reason for this lies in the oscillating character of the correlations which prevents the divergence of the fluctuations.

3.) I have proven that the probability density functional of height configurations over equatorial circles of an Edwards-Wilkinson type surface defined on a spherical substrate can be given through the effective action of the Gaussian $1/f$ noise.

4.) I have determined the von Neumann entropy of a block of L spins of an XX chain which is constrained to carry an energy current. It can be seen, that the logarithmic asymptotics present in equilibrium is preserved by the current, however the prefactor of the logarithm is two times the equilibrium value. The value of the prefactor is seen to be closely related to the structure of the excitation spectrum (i.e. the number of the „Fermi-seas”).

5.) I have shown that at a special value of the energy current, where the drastic change of correlations indicates a phase transition point, the asymptotics of the entropy is the same as in equilibrium. In the vicinity of these transition points the entropy is shown to display a special type of finite size scaling.

6.) I have shown that the weights of the quantum fluctuations in an entangled block of an XX spin chain can be described approximately as the weights of a Gibbs-Boltzmann distribution with some properly defined effective temperature. With different definitions for the effective temperature, the asymptotics was calculated and to leading order the same result was found.

3. Conclusions

I consider the following conclusions of my PhD thesis to be the most important ones:

- I have shown on the example of the transverse Ising chain, that long range, algebraically decaying correlations not necessarily result an effectively critical state, since the distribution function of the order parameter was found to be Gaussian. Moreover, the width of the Gaussians decrease with increasing flux, therefore somewhat surprisingly, the energy flux makes the system more rigid. This result further confirms the importance of the scaling functions in the studies of nonequilibrium phase transitions.

- The current-carrying steady states of the XX chain were investigated from the aspect of entanglement properties, and it was found that currents result the enhancement of the block entanglement

- It was argued, that an entangled block in an XX spin chain can be described by the introduction of an effective temperature. The simple analogies that were used in the definitions might make it possible to extend the concept of the effective temperature to the current-carrying chain, and might shed light on the possible role of an effective temperature in systems far from equilibrium.

4. List of publications that provide the basis of the theses of the dissertation

1. Viktor Eisler, Zoltán Rácz, and Frederic van Wijland, *Magnetization distribution in the transverse Ising chain with energy flux*, Phys. Rev. E, **67**, 056129 (2003).

2. Viktor Eisler and Szilárd Farkas, *Edwards-Wilkinson surface over a spherical substrate:* $1/f$ *noise in the height fluctuations*, J. Phys. A: Math. Gen. **37**, 2573 (2004).

3. Viktor Eisler and Zoltán Zimborás, *Entanglement in the XX spin chain with an energy current*, Phys. Rev. A, **71**, 042318 (2005).